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On the Stability of Okun's Coefficient in the Presence of Boom or Slump

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Abstract
The current debate in Okun's law is whether or not it responds asymmetrically to changes in business cycle. An important pre-requisite to this debate is the accurate determination of turning point dates. Two basic approaches have been employed in Okun's literature to delineate phases of recession and recovery; namely, the Bry and Boschan (BB) algorithm and the Markov switching model (MSM). This study differentiates among the turning point dates obtained through BB algorithm, MSM and National Bureau of Economic Research (NBER), and investigate how much these dates affect the conditional effect of the business cycle on Okun's law. Our results showed that using NBER and BB dates, Okun's coefficient varies across business cycle and is stronger during periods of economic recession than expansion by 11.32% and 16.96%; respectively. The result is however much different for MSM dating process: Though growth rate still exerts a negative influence on the labour market, MSM did not identify enough pronounced recession episodes to conclude that the relation between unemployment rate and growth is conditional on the business cycle. By and large, regardless of the dating structure, growth exerts a negative influence on unemployment rate whether in recession or in expansion.

Key Words: Real GDP, Unemployment rates, US, Markov switching, NBER announcement, Interactive model

JEL codes: C13, E24, E32

1. Introduction

Okun's law named after Arthur Okun was established in 1962. Its major apppeal is its ability to describe two very important macroeconomic variables of unemployment rate and economic growth in very simple and parsimonious model. In theory, Okun's law links the aggregate supply curve with Phillip's curve; and in applications, it serves as a rule of thumb for economists and policy makers. The following equation describes the simplest form of Okun's law:

$$\Delta U_t = c + \beta G_t + \varepsilon_t, \qquad t = 1,2,\cdots,T \qquad (1)$$

where ΔU is the change in unemployment rate and G represents real output growth (measured as the change in the log of real GDP). c and β are the parameters of the equation. ε is the random error term. $\beta(<0)$ is usually referred to as Okun's coefficient, while the ratio $-\frac{c}{\beta}$ measures the

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extent of growth an economy must achieve to maintain a stable rate of unemployment (Knotek, 2007). Of course, $\beta < 0$ implies that while growth rate is falling, unemployment rate is rising, and vice versa. A measure of the rate of unemployment when growth is zero is given in the intercept c.

A major shortcoming in Okun's empirical literature is that most previous studies assumed a symmetric relationship between unemployment rate and output growth whereas there are several evidences to support the claim that unemployment rate responds asymmetrically to expansions and contractions in output. For instance, Courtney (1991) identified three basic sources of asymmetry in Okun's law, namely, labour force participation, fluctuations in multi-factor productivity and changes in the distribution of sectoral growth rates. In addition, asymmetry may also result from asymmetric adjustment costs and job mismatch (Harris and Silverstone, 2001). Acemoglu and Scott (1994), Lee (2000), Harris and Silverstone (2001), Cuaresma (2003), Holmes and Silverstone (2006), Knotek (2007), Owyang and Sekhposyan (2012), and Chinn et al. (2014) have all provided empirical evidences to refute the stability of Okun's law with respect to varying business cycle.

A notable exception to Okun's critics is Ball et al. (2013) who claimed that Okun's law did not break down at any period from 1948-2011; and that the law was still stable and strong in most countries though there were pockets of variations in 7 out of 20 advanced economies that were examined. They claimed further that the argument of 'jobless recoveries' is flawed; citing the 'zero bound that has constrained monetary policy' as the reason that the end of recession did not immediately lead to full recovery of employment. However, they were not able to explain away the influence of the great recession of 2008 to satisfaction. Even after incorporating cross-country differences, the business cycle effect persisted.

A crucial pre-requisite to investigating Okun's stability in varying business cycle is accurate determination of the state of the business cycle. In Okun's literature, two basic approaches have been employed to delineate phases of recession and expansion: the Bry and Boschan (1972) algorithm (otherwise known as the Harding and Pagan approach (see Owyang and Sepkhposyan, 2012 and Cazes et al., 2013)), and the Hamilton's (1989) Markov switching model (MSM). The Bry and Boschan (BB) algorithm is a computer program designed to reproduce the NBER chronology in an automatic way while avoiding some of its shortcomings. The algorithm which was originally developed for monthly data was later modified for quarterly data by Harding and Pagan (2002). On the other hand, the Markov switching technique models contractions and expansions in a nonlinear manner following a Markov process. Owing to its nonlinear features, the model has been adopted severally in academic studies (Chauvet and Hamilton, 2005). Studies such as Acemoglu and Scott (1994), Bodman (1998) have relied on the Hamilton's (1989) technique to determine the state of business cycle in their analysis on labour markets.

Though Hamilton (1989) has claimed that his model provides turning point dates that correspond "extremely closely" to National Bureau of Economic Research (NBER) announcements however, taking a closer look at Table II (Hamilton (1989), page 374), one may observe that this claim is not exactly true. As a matter of fact, in 5 cases out of 7, MSM identified peaks that were more or less two quarters away from NBER peaks. Our observation has been noted by earlier studies like Hess and Iwata (1997) who showed that nonlinear models such as MSM did not perform better than a simple ARIMA (1,1,0) model in reproducing business cycle features.

Harding and Pagan (2002) also demonstrated that modeling the real GDP of US, UK, and Australia using random walk with drift captures the major business cycle features of the respective countries well. In a later development, the authors (Harding and Pagan, 2003) showed that based on criteria such as simplicity, transparency, robustness, and replicability, BB algorithm is superior to Hamilton's MSM in determining turning points in business cycles. Ahking (2013) also made similar submission in his paper. A salient question then is, given that accurate determination of the turning point dates is germaine to the analysis on the effect of business cycle on Okun's law, are these discrepancies not strong enough to alter previous results on the stability of Okun's law? This study therefore presents a re-appraisal of the asymmetric effect of business cycle on Okun's law, with due regard to the dating processes. Specifically, we condition the growth and unemployment series on the state of the business cycle obtained from Bry and Boschan algorithm and Hamilton's (1989) Markov switching process, and compare the results with those from NBER announcements in a simple interactive regression model.

Three major advantages accrue from this extra work; (i) a comparative analysis of the effect of various dating techniques on the asymmetric effect of growth on unemployment; (ii) a robustness check on studies such as Acemoglu and Scott (1994) and Bodman (1998); and (iii) a confirmation or otherwise of the popular claim by Hamilton (1989) that MSM dating structure provides a quantitative algorithm that can be used to identify turning points identical to NBER announcements. In addition, the Markov switching model employed here also presents an up-to-date, comprehensive, probabilistic information on the US business cycle. Finally, the proposed interactive model provides holistic approach to study the relationship between growth and unemployment rates under varying states of growth rates. Consequently, the properties of Okun's estimate, such as the magnitude, direction, stability, and so on, may be studied while the system is in boom and slump.

At this point, it is noteworthy that Okun's law has received two basic specifications – the difference and the gap models. And according to Lee (2000), Okun's estimate is sensitive to the specification that is adopted. This study adopted the difference approach for the following reasons: In gap specification, it is known that both potential output and the natural unemployment rate are not directly observable and therefore require strong (and sometimes controversial) assumptions regarding their definition and computation, such that different Okun's coefficient and consequently, business cycles may be identified depending on the selected filtering methodology (Knotek, 2007). Besides, the difference version of Okun's law is technically simple, easy to interpret and directly applicable to the original data. In addition, the difference version provides a convenient way to achieve stationarity when the unemployment rate and real GDP series contain unit root (Canarellla and Miller, 2016).

Overview of the study is as follows: Section 2 describes the methods of Markov switching and interactive regression modeling, together with the Bry and Boschan algorithm. Section 3 presents empirical results, and the last section concludes the study.

2. Data and Methods

Samples include quarterly data from 1948Q1-2017Q2. Data on US unemployment rates were obtained from US Bureau of Labor Statistics and those on real GDP from Federal Reserve Bank of St. Louis. In line with Equation (1), growth rates G_t were computed as the difference in the log

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of real GDP, while ΔU_t is the change in unemployment rate. Figure 1 displayed the quarterly plot of the two rates. The two series, in general, appear to be inversely related. It is obvious that as one goes up, the other comes down. In fact, it can be said that one series is (almost) a mirror image of the other, such that one could be used to explain, to a large extent, the variations observed in the other.

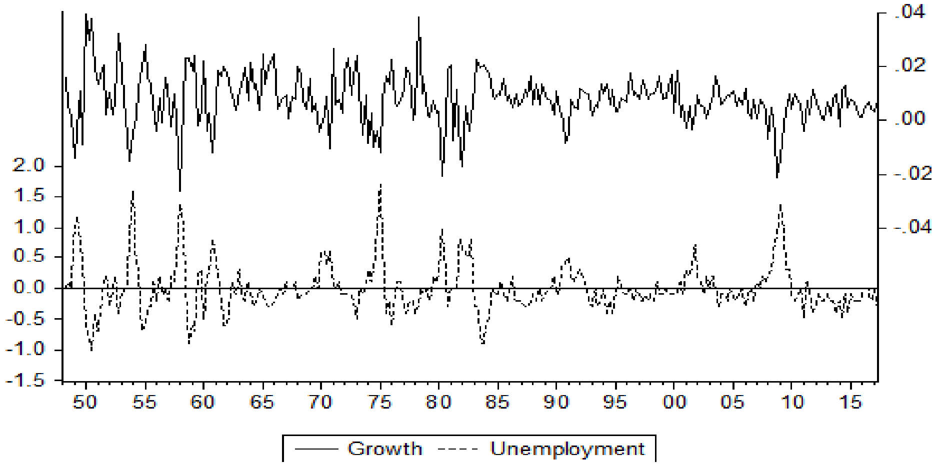

Figure 1: Quarterly Plot of US Rates of ΔU_t and $G_t (= \Delta \log(RGDP))$, 1948Q1- 2017Q2

Next we explain briefly the procedure of estimation and inference involved in the models to be employed in the study, that is, Markov switching model, the Bry and Boschan (1971) algorithm and the interactive model.

2.1 The Markov Switching Model

Switching models are models used to describe time series properties in different regimes. Models in which switching among regimes follows a Markov process are called Markov switching models (MSM). MSM was introduced by Hamilton (1989) and has been widely applied in econometrics. A brief technical detail on the model is provided below. More useful details can be found in Hamilton (1994) and Kim (1994).

For series G_t, and parameters μ and σ, consider the following system,

$$\left.\begin{aligned} &G_t = \mu_{S_t} + \varepsilon_t, \\ &\varepsilon_t \sim i.i.d.\ N\left(0, \sigma^2_{S_t}\right), \\ &S_t = 1,2,\cdots,k; \quad t = 12,\cdots,T \end{aligned}\right\}. \tag{2}$$

System (2) represents the simplest model with switching features. For *k = 2* for instance, the intercept μ takes two different values representing the expectations μ and the variances σ^2 in the 2 different states. That is, $G_t \sim N\left(\mu_1, \sigma_1^2\right)$may represent the distribution of G_t during recession while $G_t \sim N\left(\mu_2, \sigma_2^2\right)$ represents the distribution of G_t during expansion. S_t is the unobservable Markov-switching variable which evolves according to transition probabilities *P*:

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}; \tag{3}$$

where $P_{ij} = \Pr(S_t = j | S_{t-1} = i)$ and

$$\sum_{j=1}^{2} P_{ij} = 1;\ j = 1,2;\ \text{and for all } i. \tag{4}$$

Working with Chauvet and Hamilton's (2005) assumption that "recession distribution has a standard deviation very similar to that for the expansion distribution" implies for System (2) that the distribution of G_t during recession and expansion periods can be represented as

$$G_t \sim N(\mu_1, \sigma^2)$$
and
$$G_t \sim N(\mu_2, \sigma^2), \tag{5}$$

respectively. Since the states S_t are not known, the conventional maximum likelihood method may not be applicable. Consequently, Hamilton's filter discussed in details in Hamilton (1994) may be used.

Estimate of the smoothing probabilities $\hat{P}(S_t = j | \vartheta_T)$, (where ϑ_T is the information available up to time *T*) for the full sample *T* can be computed using a smoothing iterative process (Kim, 1994). Further, compute the expected duration $E[D]$ of regimes $j = 1,2$ as (Kim and Nelson, 1999)

$$E[D] = \sum_{j=1}^{\infty} jP(D = j) \approx \frac{1}{1 - P_{jj}}; \tag{6}$$

where *D* is the duration of state *j*.

To verify whether or not growth rate switched from recession to expansion, and vice versa, a natural approach is to test the following null hypotheses:

$$H_0 : \mu_1 = \mu_2. \tag{7}$$

That is, under H_0, the two states of recession and expansion are not significantly different. Since all the parameters of System (2) (incorporating assumption (5)) are identified under H_0, the Wald's test statistic may then be employed to test the null hypothesis of interest. The corresponding Wald statistic, *W* is

$$W = \frac{(\hat{\mu}_1 + \hat{\mu}_2)^2}{Var[\hat{\mu}_1] + Var[\hat{\mu}_2] + 2Cov[\hat{\mu}_1, \hat{\mu}_2]} \quad \sim \quad \chi^2_{(1)}. \tag{8}$$

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2.2 The Bry and Boschan (BB) Algorithm

Define $\alpha_t(\beta_t)$ as binary variables taking value 1 when there is a trough (peak) at *t* and *0* otherwise; so that

$$\alpha_t = 1(x_t > x_{t\pm1}, 1 \le i \le k), \qquad (9)$$
$$\beta_t = 1(x_t < x_{t\pm1}, 1 \le i \le k); \qquad (10)$$

where x_t is the predefined series. BB algorithm rests on three basic conditions; viz, (i) set $k = 5$; smooth series x_t and obtain initial set of turning points using Equations (9) and (10); (ii) eliminate enough of the turning points in (i) so that each phase of a cycle, that is peak to trough (or trough to peak), exceeds 5 months in duration, while completed cycle, that is peak to peak (or trough to trough) is at least 15 months long; and (iii) ensure that peaks and troughs alternate - if two peaks occur consecutively, one may choose the peak with the higher value of x_t.

BB algorithm was originally developed for monthly data, later Harding and Pagan (2002) proposed a modification for quarterly; this they referred to as BBQ. In contrast to BB, BBQ does not require smoothing. In addition, condition (ii) is modified so that a complete cycle exceeds 5 quarters and each phase of a cycle is at least 2 quarters long. It also sets k = 2. More details can be found in Bry and Boschan (1971) and Harding and Pagan (2002).

2.3 The Interactive Model

One simple way to incorporate conditional relationship into equation (1) is given in the following model:

$$\Delta U_t = c + \beta_G G_t + \beta_B B_t + \beta_{GB} G_t B_t + \varepsilon_t, \qquad t = 1,2,\cdots,T \qquad (11)$$

where $G_t B_t$ is the multiplicative term measuring the interaction in the data, and β_{GB} is its coefficient. β_G and β_B are the conditional regression coefficients.

Estimation procedure of βs proceeds using the conventional OLS. However, as Kam and Franzese (2007) cautioned, we must note that interpretation of regression coefficients and hypotheses testing are a little different under the interactive model. The conditional effect of growth rates on unemployment rate, for instance is given by

$$\frac{\delta \Delta U_t}{\delta G_t} = \beta_G + \beta_{GB} B_t, \qquad (12)$$

such that when the system is in recession, i.e. $B_t = 0, \left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{B_t=0} = \beta_G$, and $B_t = 1, \left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{B_t=1} = \beta_B + \beta_{GB}$ during expansion. Thus the effect of growth on unemployment rate is expected to vary according to the business cycle B_t if $B_t \neq 0$.

Accordingly, the conditional hypotheses to be tested include the following:

(i) Is the effect of growth on unemployment rate conditional on the business cycle? - $H_0 : \beta_{GB} = 0$;

(ii) Effect of growth on unemployment rate - $H_0 : \beta_G = \beta_{GB} = 0$;

(iii) Effect of business cycle on unemployment rate - $H_0 : \beta_B = \beta_{GB} = 0$;

(iv) Model adequacy: Does unemployment rate depend on growth, business cycle or some combination of both? - $H_0 : \beta_B = \beta_G = \beta_{GB} = 0$.

Since all the parameters of Model (11) are known under H_0; standard test statistics may be used to test hypotheses (i) to (iv).

3. Results and Discussion

Here we investigate the conditional effect of recessions and/or recoveries on Okun's law using an interactive model described earlier in section 2.3. We introduce a multiplicative term in Okun's equation to measure the direct dependence or otherwise of unemployment-growth link on business cycle. Relevant equations have been stated in System (11). As a prerequisite to the analysis, we must first determine the turning point dates. This will be achieved using the Markov switching method and Bry and Boschan algorithm explained in sections 2.1 and 2.2, respectively. Our intent is to compare the outcome from MSM and BB turning point dates with that of NBER. This is crucial since accurate determination of the turning point dates is germaine to the analysis of the conditional effect of the business cycle on Okun's law.

3.1 US Business Cycle Analysis

Table 1 refers to the maximum likelihood estimates (MLEs) of the parameters corresponding to System (2) and assumption (5). We observed as follows:

The Markov switching regression model classified the growth rate series into two distinct regimes in accordance with Figure 1. On the average, US growth rate declined quarterly by 1.1% in State 1, while state 2, the high growth state corresponded to a quarterly growth rate of 0.88%: The common volatility level stood at 0.76% for the entire period.

We immediately tested the null hypothesis of equality of the two regimes as presented in Equation (7). The Wald statistic W (See Equation (8)) is 24.54131 (p = 0.000). It is therefore evident that US growth rate system underwent two distinct states of recession and expansion during 1948Q1 - 2017Q2.

Table 1: MLEs of MSM for Business Cycle Analysis of US Economy

$$G_t = \mu_{S_t} + \gamma G_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2), S_t = 1,2; \quad t = 1,2,\cdots,T$$

Parameter	μ_1	μ_2	σ	γ^+
Estimate	-0.011007	0.008766	0.00762	0.434921
Std. err.	0.004549	0.001073	0.0004	0.063814

Standard errors in parentheses. $^+$ lag 1 of growth rate included to correct for serial correlation.

Estimated transition probability matrix,

$$\hat{P} = \begin{pmatrix} 0.300634 & 0.699366 \\ 0.039831 & 0.960169 \end{pmatrix}$$

indicated that US growth rate system is less likely to remain in recession, $\hat{P}_{11}$ being 30.1% compared to $\hat{P}_{22}$ which is 96.01%. This submission is in line with the quarterly growth rate plot in Figure 1 as the growth rate system entered into recession only twice since 1991 - briefly in 2001 and then in 2008; otherwise it has been in state 2 in the recent past. It was not a surprise therefore that the transition probability, $\hat{P}_{22}$ of the non-recession state is 96.01%. This finding is further confirmed by the expected duration results, as the estimated expected length of stay in state 1 is approximately 1.4 quarters as opposed to the non-recession state which is approximately 25.1 quarters. Also from the probability estimates we infer that transition from the expansion into recession state is less likely, only a 4% chance, whereas, movement from recession to expansion is more likely with approximately 70% probability. This is expected since the two very pronounced periods of expansion, that is, 1950Q1 and 1978Q2 were preceeded by recession periods 1949Q4 and 1975Q1, respectively (See bottom panel of Figure 2). Interestingly, out of these two pronounced expansion periods, only the episode of 1978Q2 was followed by a recession in 1980Q1: giving supporting evidence that the likelihood of US economy transiting from recession to expansion is higher than the other way around.

The smoothed probability plot of the recession periods is displayed in the upper panel of Figure 2. Comparing the recession smoothed probability graph with US economy quarterly plot, it is evident that based on the real GDP series, the Markov switching model employed here extracted the recession periods accurately. Usually, researchers assume that switching occurred when smoothing probability is greater than 0.5. Thus, the turning point dates are: 1949Q4-1950Q1, 1958Q1-1958Q2, 1960Q4, 1970Q4-1971Q1, 1980Q2-1980Q4, 1981Q2-1982Q2, and 2008Q4-2009Q2. We note, in passing, that the economy went into recession, (though not well pronounced) in 1953Q3 - 1954Q1, 1973Q3 - 1975Q2, 1977Q4 - 1978Q2, 1991Q1 and 2001Q1 - 2001Q4 as indicated in the smoothed plot. However, these periods were not pronounced enough to be classified as significant recession periods as their smoothing probabilities were less than 0.5. Accordingly, the downward swings at those periods, as shown in the quarterly growth plot, were very short and sharp.

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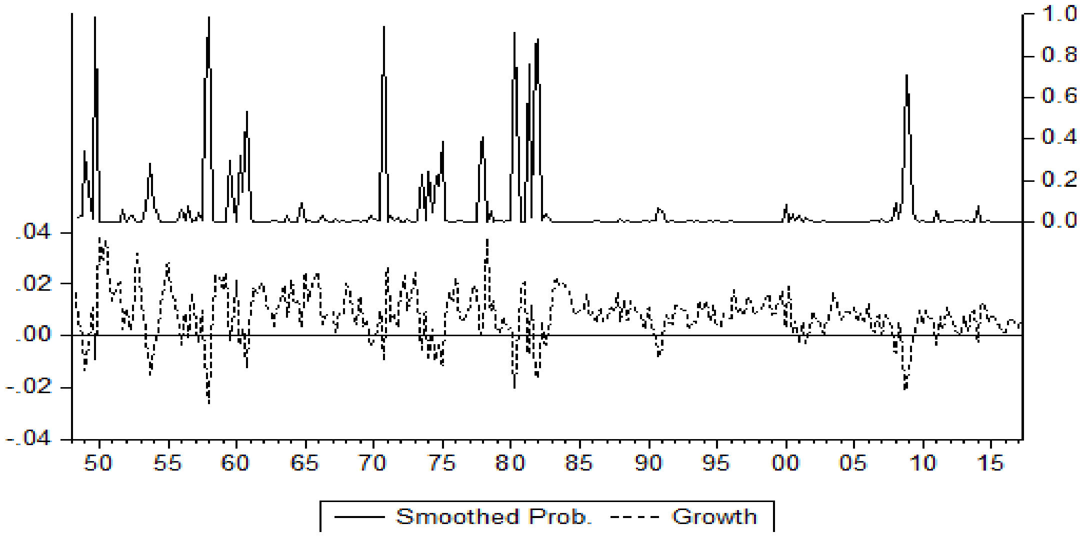

Figure 2: Growth Plot and Recession-Smoothed Probability Plot

3.2 Comparing Turning Point Dates by Methods

This section compares the turning point dates obtained from MSM and BB with those from NBER. Table 2 displayed turning point dates from NBER, MSM and BB. The first column contained NBER chronology of the recession periods extracted from www.nber.org/cycles.html. We note that the National Bureau of Economic Research (NBER) Business Cycle Dating Committee is a private not-for-profit organization responsible for the announcements of the turning point dates. NBER chronology is determined using Burns and Mitchell's (1946) methodology. NBER dating has been adopted among researchers as the standard for delineating recession and expansion periods and is often used as basis for comparison on business cycle (Ahking, 2013). Column 2 of Table 2 contained MSM cycle dates obtained from the smoothed probability plot displayed in the top panel of Figure 2. The last column referred to BB dates reproduced from Table 5, Ahking (2013).

Table 2: NBER Versus MSM and BB Dates

NBER		*MSM*		*BB*	
Peak	Trough	Peak	Trough	Peak	Trough
1948Q4	1949Q4	1949Q4	1950Q1	1948Q4	1949Q2
1953Q3	1954Q2	Not pronounced		1953Q2	1954Q1
1957Q3	1958Q2	1957Q4	1958Q2	1957Q3	1958Q1
1960Q2	1961Q1	1960Q4	1960Q4	1960Q1	1960Q4
1969Q4	1970Q4	1970Q4	1971Q1	1969Q3	1970Q1
1973Q4	1975Q1	Not pronounced		1973Q4	1975Q1
1980Q1	1980Q3	1980Q2	1980Q4	1980Q1	1980Q3
1981Q3	1982Q4	1981Q2	1982Q2	1981Q3	1982Q1
1990Q3	1991Q1	Not pronounced		1990Q2	1991Q1
2001Q1	2001Q4	Not pronounced		Not found	
2007Q4	2009Q2	2008Q4	2009Q2	2007Q4	2009Q2

NBER turning point dates were extracted from www.nber.org/cycles.html.

Figure 3 plots the smoothing probability of a recession superimposed with NBER chronology - the shaded areas are NBER calculations of the recession periods. In agreement with

Ahking (2013), one may observe some discrepancies between NBER and MSM dates; for instance in 1953Q3 - 1954Q2, 1973Q4-1975Q1, among others, where the smoothing probabilities from MSM were less than *0.5* and consequently, those periods were not included as recession episodes in MSM dating structure, though the periods were announced by NBER. Moreover, observe that the peaks from MSM did not coincide with any of NBER's. On the other hand, we observed that BB algorithm identified all NBER recession dates except 2001Q1 as opposed to MSM which found 7 out of 11. In addition, the peaks identified by BB algorithm coincided with NBER's in 7 out of 11 cases whereas none of those of MSM did. We figured that the discrepancies in MSM could be explained in the words of Harding and Pagan (2002) that the Markov switching model produced cycles that are too extreme, especially with respect to the cumulative movements of the cycles, that is, the cumulated output losses from peak to trough of a business cycle. A salient question to ask is whether or not these discrepancies are strong enough to affect the result of the investigation on the conditional effect of the business cycle on Okun's law. This we intend to find out in what follows.

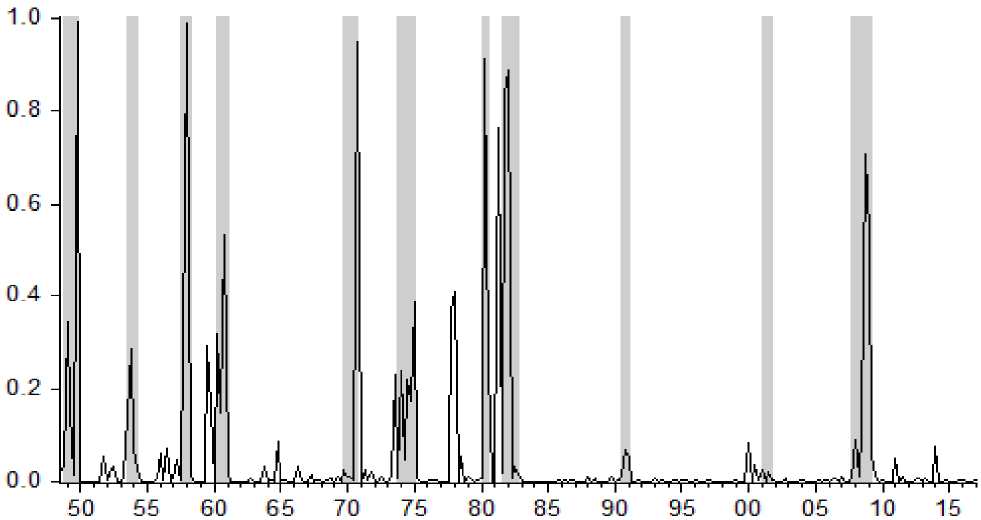

Figure 3: Smoothed Probability of a Recession
Notes: The shaded areas show the recessionary periods calculated by the NBER.

3.3 Interactive Regression Results

In this section, the turning point dates displayed in Table 2 were utilized to create an indicator variable B_t which was incorporated in an interactive regression model as explained in section 2.3. Parameters in System (11) were then estimated to yield the results displayed in Table 3. Breakdown of the hypotheses testing outcomes also as earlier highlighted in section 2.3 can be found in Table 4.

Now, is the relation between unemployment rate and growth conditional on the business cycle? Using NBER dates, we note that $\hat{\beta}_{GB} = 0.113212$ with a standard error of 0.038 corresponded to a p-value of 0.0028. This tells us that when NBER announcement is utilized as turning point dates, unemployment-growth link is conditional on business cycle to an extent of 11.3%. The same inference can be drawn in the case of BB algorithm since $\hat{\beta}_{GB} = 0.169632$ with a p-value of *0.000*. For MSM, though the test $H_0 : \beta_G = \beta_{GB} = 0$ produced a Wald statistic *W = 172.5327* (*p = 0.000*) which strongly supports the claim that using MSM dating process, growth rate still exert a negative influence on the labour market. However same cannot be said for the conditional effect of business cycle on Okun's law: $\hat{\beta}_{GB} = 0.032646$ with a standard error of 0.037042 corresponded to a p-value of 0.3789. This implies that for the period 1948Q1 - 2017Q2, the Hamilton's Markov discrete state model did not identify enough pronounced recession episodes to conclude that the relation between unemployment rate and growth is conditional on the business cycle.

Next we ask, how much does growth affect unemployment rates under varying business cycle? Under NBER dating structure, the conditional coefficient of growth

during recession, $\left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{B_t=0} = \beta_G = -0.239$; and

during expansion, $\left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{B_t=1} = \beta_B + \beta_{GB} = \text{-}0.239 + 0.113 = \text{-}\,0.126$.

Table 3: OLS Estimates for Interactive Model

$$NBER^+ : \quad \Delta U_t = c + \beta_G G_t + \beta_B B_t + \beta_{GB} G_t B_t + \gamma_1 \Delta U_{t-1} + \gamma_2 \Delta U_{t-3} + \varepsilon_t ,$$
$$MSM^+ : \quad \Delta U_t = c + \beta_G G_t + \beta_B B_t + \beta_{GB} G_t B_t + \gamma_1 \Delta U_{t-1} + \varepsilon_t ,$$
$$BB^+ : \quad \Delta U_t = c + \beta_G G_t + \beta_B B_t + \beta_{GB} G_t B_t + \gamma_1 \Delta U_{t-1} + \varepsilon_t ,$$

Parameter	*Interactive*		
	NBER	BB	MSM
c	0.347635 (0.034169)	0.272430 (0.038302)	0.251652 (0.058863)
β_G	- 0.239728 (0.032068)	- 0.314762 (0.035707)	- 0.236304 (0.032075)
β_B	- 0.314285 (0.042156)	- 0.197933 (0.044141)	- 0.101152 (0.061453*)
β_{GB}	0.113212 (0.037530)	0.169632 (0.039818)	0.032646 (0.037042*)
γ_1	0.384361 (0.039801)	0.426217 (0.036211)	0.440110 (0.040626)
γ_2	- 0.082128 (0.035601)	NA	NA
Adjusted R^2	0.712802	0. 691656	0. 648083
LogL	40.10801	31. 04000	12. 79930
BIC	-0.169844	-0. 123109	0. 009070
Q (10)	16.791 (0.079)	17.481 (0.064)	15.961 (0.101)
ARCH(10)	13.50327 (0.1969)	17.52792 (0.0635)	16.33818 (0.0904)

$^+$ means some lag terms of ΔU_t were included to avoid serial correlation.

Standard errors in parentheses. p-values in parentheses for Q(10) and ARCH(10).

* means not significant at 0.05. NA means Not Applicable

This implies that a unit decrease in the growth rate during recession would bring about a 23.9% increase in the unemployment rate whereas a unit increase in growth during expansion would reduce unemployment rate by 12.6%. According to Table 4, the null hypothesis $H_0 : \beta_G = \beta_{GB} = 0$ has Wald statistic *W = 98.49496* with *p = 0.000.* Thus the NBER-regression results agreed with Cuaresma (2003), Knotek, (2007) and Owyang and Sekhposyan (2012), among others, that the relationship between unemployment rate and growth for the US economy varies across business cycle and is stronger during periods of economic recession than expansion. Similarly, with respect to the ratio, $-\frac{c}{\beta}$, the minimum growth rate required to maintain stable economy during recession is 1.450 while that of expansion is 2.748.

Table 4: Hypothesis Testing for Interactive Model

Test	*Statistic* NBER	BB	MSM
$H_0 : \beta_{GB} = 0$	9.099770 (0.0026)	18.14909 (0.0000)	0.776758 (0.3781)
$H_0 : \beta_G = \beta_{GB} = 0$	98.49496 (0.0000)	135.4095 (0.0000)	172.5327 (0.0000)
$H_0 : \beta_B = \beta_{GB} = 0$	63.00806 (0.0000)	42.47003 (0.0000)	3.657475 (0.1606)
$H_0 : \beta_B = \beta_G = \beta_{GB} = 0$	270.8865 (0.0000)	257.0798 (0.0000)	191.6952 (0.0000)

In the same vein, for BB algorithm, the conditional coefficient of growth during recession and expansion were -0.1979 and -0.0283, respectively. In other words, output growth has greater effect on unemployment rate during recession than expansion with a difference of 16.96%. Similarly, given BB dating structure, the minimum growth rate required to maintain stable economy during recession is 1.376 while that of expansion is 9.626. For MSM, we cannot discuss the conditional effect of growth on unemployment rate since we have earlier rejected the hypothesis that Okun's law is conditional on the business cycle.

In addition, a measure of how much business cycle affects unemployment rate in US economy is summarized below: For NBER dates,

at minimum growth rate, $\left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{G_t=-0.02623} = \beta_B + \beta_{GB} \times -0.02623 =$ -0.31725,

at mean growth rate, $\left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{G_t=0.007754} = \beta_B + \beta_{GB} \times 0.007754 =$ -0.31341,

at maximum growth rate, $\left.\frac{\delta \Delta U_t}{\delta G_t}\right|_{G_t=0.03908} = \beta_B + \beta_{GB} \times 0.03908 =$ -0.30986.

That is, while growth is at minimum, the varying business cycle has a negative effect on unemployment rate under NBER dating structure; and the effect increases with increasing growth rate. In the same vein, under BB algorithm, while growth rate is minimal, average or at a maximum level, the conditional effect of business cycle on unemployment rate is approximately -0.2024, -0.1966 and -0.1913, respectively.

Further, the relative performance of the three models were compared using the adjusted R^2 statistic, the loglikelihood function (*LogL*) and the Bayesian Information Criterion (*BIC*). Adjusted R^2 statistic indicated that the amount of variation in the unemployment rate being explained by growth under varying business cycle remains high, ranging from 64.8% to 71% with NBER leading the lot followed by BB then MSM. In addition, NBER and BB dating structures had higher loglikelihood and lower *BIC* values, indicating that these two models performed better than the MSM-interactive model.

Lastly, the specification diagnostics showed that after the inclusion of appropriate lag terms (using the correlogram as guide), the residuals from the three interactive models did not exhibit additional serial correlation or heteroscedasticity, as measured by Q(10) and ARCH(10), respectively (where Q(10) is the Ljung-Box Q-statistic for standardized residuals and ARCH (10) the autoregressive conditional heteroscedasticity test statistic, at lag 1 through 10).

4. Conclusion

A very essential pre-requisite to determining the conditional effect of varying business cycle on Okun's law is to obtain accurately the turning point dates of recessions. This study re-examined the conditional effect of business cycle on the relation between unemployment and growth rates with due regard to the dating process. We differentiated among the turning point dates obtained through National Bureau of Economic Research (NBER), Bry and Boschan (BB) algorithm and Markov switching modeling technique, and investigated how much they affect the conditional effect of the business cycle on Okun's law. Empirical results showed that using NBER and BB dating structures, Okun's coefficient varies across business cycle and is stronger during periods of economic recession by 11.32% and 16.96%, respectively. Specifically, we found that for NBER dates, a unit decrease in the growth rate during recession would bring about a 23.9% increase in the unemployment rate whereas a unit increase in growth during expansion would reduce unemployment rate by 12.6%.

Similarly from BB-regression result, we observed that a unit decrease in the growth rate during recession would bring about a 19.8% increase in the unemployment rate whereas a unit increase in growth during expansion would reduce unemployment rate by 2.8%. As a direct consequence, the minimum growth rate required to maintain stable economy during recession and non-recession periods were 1.455 and 2.759, respectively for NBER and 1.376 and 9.626, respectively for BB algorithm. The result for MSM dating process is however much different: Though growth rate still exerts a negative influence on the labour market, the Hamilton's (1989) Markov discrete state model did not identify enough pronounced recession episodes to conclude that the relation between unemployment rate and growth is conditional on the business cycle. By and large, regardless of the dating structure, growth exerts a negative influence on unemployment rate whether in recession or in expansion. We also reported that adjusted R^2 value did not indicate any break down in the ability of growth rate to predict unemployment rate either in recession or expansion period as the statistic remained high in the entire period under investigation.

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Finally, a note of caution: One major problem that usually arises when comparing results on Okun's law is that several versions exist in the empirical literature. In particular, Okun's law has received two basic specifications - the difference and the gap models. According to Lee (2000), Okun's estimate is sensitive to the specification that is adopted. Besides, sample period and frequency of the data are also two important factors that could alter Okun's estimate. Thus results from studies on Okun's law should be reviewed bearing in mind this note of caution.

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