Journal of Applied Economics and Business Research
JAEBR, 10(4): 221-233 (2020)

Chaotic Analysis of the Real Estate Investment Trusts Index Returns: An Application of the Largest Lyapunov Exponent

Emmanuel Anoruo[1]
Coppin State University, USA

Abstract
This paper examines the presence of chaotic dynamics in Real Estate Investment Trusts (REITs) index returns for the United States. Specifically, the paper uses the largest Lyapunov exponent to explores the chaotic behaviors of all, mortgage and equity REITs index returns for the time period running from January 1980 through December 2018. The sample is divided into two (pre-crisis and post-crisis periods) to ascertain the impact of the U.S. subprime mortgage crisis on the dynamic structure of REITs index returns. The ADF, Phillips-Perron and the KPSS are used to determine the time series properties of the REITs index returns. To test for nonlinearity, the paper implemented the BDS test. The results from the unit root tests indicate that the three REITs index returns have zero order of integration. The BDS test results show that the REITs index return series are nonlinear. The results from the largest Lyapunov exponent test for the full and pre-crisis sample periods provide supportive evidence of chaotic dynamics in REITs index returns. However, for the post-crisis period, the study finds that the behavior of the three REITs index returns is governed by stochastic processes as opposed to chaotic dynamics. The implications of the results are discussed.

Keywords: REITs, Lyapunov exponent, BDS test, chaos, nonlineari
JEL classification: C01, C58, G10

1. Introduction

This paper explores the chaotic dynamics in Real Estate Investment Trusts (REITs) index returns for the United States. In particular, the study examines the possibilities that the index returns for three REIT classes namely — all REITs, equity REITs, and mortgage REITs are nonlinear and chaotic processes. According to the National Association of Real Estate Investment Trusts (NARIT), REITs comprise companies that either own or finance income-generating real estate across a spectrum of property sectors. REIT companies are required to pay out at least 90 % of their taxable incomes to their shareholders. However, it is not uncommon for some of these

[1] Correspondence to Emmanuel Anoruo, Email: eanoruo@coppin.edu

companies to payout 100% of their taxable incomes to their shareholders. Shareholders are in turn obligated to pay income taxes on the dividends they receive from the REIT companies.

Investors and portfolio managers tend to closely monitor movements in REIT markets as disturbances in these arenas can be easily transmitted to other markets, such as the stock, bond, oil, and real estate markets, through the spillover effect. Consequently, a clear understanding of the behaviors of REITs index returns is crucial to investors and portfolio managers as they develop appropriate strategies to predict movements in REIT markets. Studies in the extant literature including Jirasakuldech and Emekter (2012) BenSaïda and Litimi (2013) have documented that financial assets such as stocks, bonds and REITs are not random walk processes; they are however characterized by nonlinearities and deterministic chaos.

Jirasakuldech and Emekter (2012) suggest that REIT markets are susceptible to nonlinearity because the real estate markets which most REIT investments are predicated on, possess unique features. Some examples of these features include thin trading, illiquidity, high transaction costs, short selling, limitation of foreign ownership and imperfection in financial information disclosures. Furthermore, Liow and Webb (2008) point out that nonlinearities in REIT prices may be caused by behaviors of irrational investors, in conjunction with the interactions between various market participants. If REIT index returns were nonlinear and chaotic dynamics, they should then be modelled accordingly. Consequently, any attempt to predict the behavior of REITs index returns via linear models will unarguably produce biased inferences. REITs as investment instruments are attractive to investors because they have historically accorded their shareholders with competitive total returns through stable dividend incomes and long-term capital gains, as noted by NAREIT.

Investors frequently tout REITs as portfolio diversifiers given that they have low correlation with other financial assets such as common stocks and bonds. This feature entails that investors can reduce risk by including REITs in their portfolios. In addition, REITs provide avenues through which investors can mitigate the illiquidity problem associated with real estate assets. They therefore serve as a conduit through which the real estate markets can be integrated with the broader capital markets. Per chaos theory, a system is deemed chaotic if it possesses unique attributes such as nonlinearity, irregular periodicity and sensitivity to initial conditions. In a chaotic system, a small change in a phase space tends to expand exponentially.

This study contributes to the literature in a number of ways. First, the study utilizes an array of procedures to test for nonlinearity in REITs index returns. Second, the study implements the largest Lyapunov exponent test to ascertain the chaotic behavior of REITs index returns. Third, unlike the earlier studies, the present study utilizes longer time series on all REITs, equity REITs and mortgage REITs index returns. The largest Lyapunov exponent was adopted by the study because it is robust to both nonlinear and stochastic systems. Furthermore, the largest Lyapunov exponent can distinguish between stochastic and chaotic dynamics even in the presence of moderate noise. The results from this study provide supportive evidence that all REITs, equity REITs and mortgage REITs index returns are nonlinear and chaotic for both the full sample and pre-subprime crisis periods. However, for post-crisis period, the study finds that all REITs, equity REITs and mortgage REITs index returns are nonlinear and

 ISSN 1927-033X

non-chaotic.

The remainder of the paper is organized as follows. Following the present introduction, Section 2 provides the literature review. Section 3 discusses the methodology. Section 4 presents the data and the descriptive statistics. Section 5 discusses the empirical results. Section 6 provides the conclusions and the implications of the study.

2. Literature Review

Earlier studies on the chaotic behavior of asset returns have produced mixed results. For instance, Jirasakuldech and Emekter (2012) explore the nonlinear dynamic and chaos behavior of equity, mortgage, and hybrid REIT returns using the BDS (Brock-Dechert-Scheinkman,1996) test, the k-map and z-map model proposed by Larrain (1991), the Markov chain test advanced by McQueen and Thorley (1991) and the time reversibility test introduced by Ramsey and Rothman (1996). They find that the equity, mortgage, and hybrid REIT return series are nonlinear during both pre- and post-1993 Tax Reform Act. They concluded based on their results that the behavior of REIT returns did not changed following the 1993 Tax Reform Act. Ambrose, et al. (1992) explore the issue of market segmentation between the real estate and the capital markets using the rescaled range analysis. They find that the stock market exhibits random walk behavior. They further find that portfolios of mortgage and equity REIT returns are random walk processes. They however failed to find supportive evidence of market segmentation between the real estate and stock markets. Okunev, et al. (2000) examined the relationship between the stock market and the public real estate market. They find supportive evidence of a nonlinear unidirectional causality from the stock market to public real estate market. Based on this result, they concluded that the relationship between the real estate stock and general stock market returns is nonlinear. Liow and Webb (2008) examine the interdependences of real estate markets for the US, UK, Japan, Australia, Hong Kong, and Singapore using the BDS test. They failed to find conclusive evidence of the existence of deterministic nonlinear return behavior in these six real estate markets. They however find that the degree of co-movements between these real estate markets varied. Based on their findings, they suggested that it is important to account for nonlinearity when modeling real estate markets and their returns.

On a related study, Bhattacharya, et al. (2018) explored the behavior of foreign exchange rates of Brazil, Russia, India, China, and South Africa (BRICS) using the maximum Lyapunov exponent and Correlation Dimension procedures. They find that exchange rates for Brazil, Russia, India and China exhibit chaotic behavior. However, they failed to find supportive evidence that South African exchange rate is characterized by chaotic dynamics. Kumar and Kamaiah (2016) using the largest Lyapunov exponent test find that the foreign exchange rates for Brazil, Russia, Indian, China and South Africa are governed by chaotic structures. BenSaïda and Litimi (2013) examine the chaotic behaviors of six major stock index returns including the S&P 500, NASDAQ composite, Nikkei 225, CAC 40, FTSE 100 and Dax. They find that the stock index returns are stochastic but not chaotic dynamics. They find similar results for Euro EUR, Japanese Yen, Pound Sterling, Canadian dollar, Swedish krona, and Swiss Franc dollar exchange rates. Based on their results, they concluded that the financial markets for these countries are better modeled as stochastic processes rather than chaotic dynamics. Mishra, et al. (2011)

 ISSN 1927-033X

explored the presence of nonlinear dependence and deterministic chaos in the rate of returns series for six Indian stock market indexes using the BDS and the largest Lyapunov exponent. They find evidence of chaos in two of the six indexes in their study. They concluded that the existence of chaos in the two stock index returns suggests that profitable nonlinearity-based trading rules could exist at least in the short-run.

Webel (2012) used the 0-1 test to examine the existence of chaos in German stock returns. Webel finds supportive evidence of chaotic structures in the returns of all stocks listed on the DAX stock exchange. Tiwari and Gupta (2019) applying the 0-1 and the largest Lyapunov exponent test advanced by BenSaïda and Litimi (2013) examined the existence of chaos in G7 stock market returns. They used raw and denoised data on the stock market returns in their study. The results from the 0-1 test provided evidence of chaos in both the denoised and raw data. However, the results from the largest Lyapunov exponent test provided evidence of chaos in only the denoised return series. They suggested based on their results that it is better to denoise the data when testing for chaos in financial time series. Serletis and Shintani (2003) employing the Lyapunov exponent test explored the chaotic behavior of the US stock market returns for the time period running from January 3, 1928 through October 18, 2000. Based on the results from the Lyapunov exponent test, they find evidence against the existence of chaos in S&P 500 and the Dow Jones Industrial Average.

Diaz (2015) examined the nonlinear properties of the London gold price fixes using the BDS and the Lyapunov exponent tests. He finds that the returns of the London gold price fixes display chaotic behavior. Litimi, et al. (2019) investigate the chaotic behavior of financial market volatility indexes including the VIX (S&P 500 volatility, U.S.), VSTOXX (STOXX 50 volatility, Eurozone), JNIV (Nikkei 225 volatility, Japan), VFTSE (FTSE 100 volatility, U.K.), VDAX (DAX 30 volatility, Germany), VCAC (CAC 40 volatility, France), VAEX (AEX volatility, Netherlands), and VSMI (SMI volatility, Switzerland). Their study covers the time period spanning January 1, 2001 through December 15, 2016. They find supportive evidence of chaos in the eight daily major volatility indexes under study.

It is obvious from the preceding literature review that most of the earlier studies on the chaotic behavior of financial assets produced mixed results and also focused mainly on stock and foreign exchange markets. Studies on the chaotic dynamics in REITs index returns are sparse. To fill this gap, the present study extends the debate on the behavior of financial assets to REIT markets. Specifically, the study investigates the presence of chaotic dynamics in REITs index returns.

3. Methodology

The empirical analysis of the study commences with the application of unit root tests including the Augmented Dickey-Fuller (ADF) proposed by Dickey and Fuller (1979), the Philipps-Perron procedure advanced by Phillips and Perron (1989) and the KPSS test developed by Kwiatkowski, et al. (1992). The unit tests are undertaken to ensure that the rejection or acceptance of the null hypothesis of independently, identically distribution (IID) under the BDS procedure is not caused by the nonstationarity of the underlying financial time series. These unit root tests have been extensively used in the extant literature and as such they will not be rehearsed in this paper.

 ISSN 1927-033X

3.1. BDS Nonlinearity Test

This paper uses the BDS nonlinearity test advanced by Brock, et al. (1996) to determine the existence of nonlinear dependence in the data. The BDS test statistic is computed as follows:

$$W_m(\varepsilon,T) = \frac{\sqrt{T}[C_m(\varepsilon,T) - C_1(\varepsilon,T)^m]}{\sigma_m(\varepsilon,T)} \quad (1)$$

In equation (1), $W_m(\varepsilon,T)$ is the BDS test statistic, $\sigma_m(\varepsilon,T)$ stands for the standard deviation of $C_m(\varepsilon,T)$, m represents the embedding dimension, while ε is the maximum difference between pairs of observations considered in computing the correlation integral. The BDS technique is asymptotically normally distributed with zero mean and unit variance [i.e. $N(0,1)$]. The null hypothesis to be tested under the BDS procedure is that the residuals of the financial time series of interest are independently, identically distributed (i.i.d).

3.2. Largest Lyapunov Exponent Chaos Test

Lyapunov exponents (λ) evaluate the sensitivity to changes on the initial conditions of a dynamic system. They basically provide the average rate of convergence or divergence from disturbed initial conditions. In other words, the largest Lyapunov exponent reveals the average rate of convergence or divergence between neighboring trajectories of an orbit following some disturbance, in a given time horizon. Under the Lyapunov technique, the rate of convergence or divergence is calculated through an exponential function. The value of the largest Lyapunov exponent (λ) plays a significant role in detecting the existence of chaotic behavior of economic and financial time series. A positive value of the largest Lyapunov exponent (i.e. $\lambda > 0$) reveals the presence of chaotic behavior, which implies that adjacent trajectories diverge exponentially. On the other hand, a negative value of the largest Lyapunov exponent (i.e. $\lambda < 0$) suggests the presence of exponential convergence of trajectories to initial conditions. This implies that neighboring trajectories remain close to one another. However, a zero value of the Lyapunov exponent (i.e. $\lambda = 0$) indicates bifurcation, implying nearby trajectories neither converge nor diverge exponentially.

There are several algorithms that can be used to detect chaos including the Rosenstein (Rosensten, et al. (1995), Kantz (Kantz, 1994) and Wolf (Wolf, et al., 1985) procedures. However, this study adopts the Rosenstein method because of ease of application. It is imperative to point out that the Rosenstein technique does not directly provide the value of the largest Lyapunov exponent (ʌ), but instead it relies on the following function:

$$y(i,\Delta t) = {}^{1}/_{\Delta t}\{lnd_j(i)\}, d_j(i) = min_{xj}\|x_j - x_j'\| \quad (2)$$

In equation (2), x_j stands for a given point, while x_j' represents its nearest neighbor in the phase space. The technique is based on the relationship between d_j and the Lyapunov exponents given by:

$$d_j(i) \approx e^{\lambda(i\Delta t)} \quad (3)$$

The largest Lyapunov exponent is computed by estimating the tendency of the most linear part of the function.

3.3. Data and Descriptive Statistics

The study employs monthly data on REITs including all, Equity, and Mortgage REIT indexes. The data on REIT indexes were extracted from the website of the National Association of Real Estate Investment Trusts, Inc. (NAREIT) at https://www.reit.com/data-research/reit-indexes/monthly-index-values-returns. The study period runs from January 1980 through December 2018. Equity REITs either invest in or own real estate. While mortgage REITs invest in financial instruments that are secured by mortgages. Mortgage REITs also lend money to real estate owners and developers. All REITs are a combination of the equity and mortgage REITs. The REITs index return series are calculated as follows: $[(R_t - R_{t-1} + D_t)/ R_{t-1}]*100$, *where* R_t represents the value of the REIT index at time t, and D_t stands for the dividends between $t-1$ and t time periods. The sample period was divided into two to assess the impact of the subprime crisis which ended in 2008. The first sample (pre-crisis) period runs from January 1980 to July 2007. The second sample (post-crisis) period runs from January 2010 through December 2018.

Table 1: Descriptive Statistics (January 1980 -December 2018)

	ALLR	EQR	MOR
Mean	0.94	0.47	0.66
Maximum	27.97	30.50	17.20
Minimum	-30.23	-31.91	-24.11
Std. Dev.	4.73	4.89	5.23
Skewness	-0.83	-0.72	-0.93
Kurtosis	10.85***	11.79***	6.48***
Jarque-Bera	1254.43***	1546.49***	303.90***
Probability	0.00	0.00	0.00
Observations	468	468	468

*** indicates level of significance at the 1%. ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns

The descriptive statistics for the REITs index returns are presented in Table 1. The mean values for all, equity and mortgage REITs index returns are 0.88, 0.44 and 0.60 percent, respectively. From the minimum and maximum statistics reported in Table 1, it can be observed that the REITs index return series have fluctuated greatly for the time period under study. For instance, the index returns for equity REITs varied from a minimum of -31.91 percent to a maximum of 30.50 percent. The standard deviations reveal that among the REITs index returns, mortgage returns displayed the greatest variability (5.23 percent) from the mean. However, all

REITs index returns with a standard deviation of 4.73 percent recorded the least deviation from the mean among the three-return series. The statistics presented in Table 1 reveal that all the three REIT return series are negatively skewed. The Kurtosis statistics, 10.85, 11.79, and 6.48 respectively for all, equity and mortgage REITs index returns exceed 3, implying the presence of heavy tail in the distributions. The Jarque-Bera statistics, 1254.43, 1546.49, and 303.90, respectively for all, equity and mortgage REITs index returns reject the null hypothesis of normality at the 1 percent level of significance.

4. Empirical Results

Table 2 displays the unit root results from the ADF, Phillips-Perron and the KPSS procedures. The optimal lags for both the ADF and the Phillips-Perron unit root tests were determined by Akaike Information Criteria. The null hypothesis of nonstationarity under the ADF and Phillips-Perron unit root tests is rejected at the 1% significance level for all, equity, and mortgage REITs index returns. For example, the ADF test statistics using constant only are -9.13, -9.46, and -8.40, respectively for all, equity, and mortgage REITs index returns. These test statistics are statistically significant at the 1% level of significance. Similar results are indicated by the Phillips-Perron tests. The KPSS test results fail to reject the null hypothesis of stationarity for all the three REITs index return series. In each of the cases, the computed test statistics are less than the critical values at the conventional levels. For example, under the test with constant and trend, the KPSS test statistics 0.038, 0.028, 0.048, respectively for all, equity and mortgage REITs index returns are less than the 1%, 5%, 10% critical values. The finding of stationarity in the REITs index return series implies that the rejection of the null hypothesis of IID could not be attributed to the non-stationarity of these variables.

Table 2: Unit Root Test Results (January 1980 - December 2018)

	With Constant Only			**With Constant and Trend**		
Series	*ADF*	*P-P*	*KPSS*	*ADF*	*P-P*	*KPSS*
ALLR	-9.13^{***}(5)	-19.73^{***}(6)	0.054 (7)	-9.13^{***}(5)	-19.72^{***}(7)	0.038(7)
EQR	-9.46^{***}(5)	-20.23^{***}(5)	0.028 (6)	-9.44^{***}(5)	-20.21^{***}(5)	0.028(6)
MOR	-8.40^{***}(4)	-19.49^{***}(7)	0.056 (8)	-7.42^{***}(5)	-19.48^{***}(7)	0.048(7)

*** indicates 1% level of significance. The figures in parenthesis are the lag lengths. ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns. The 1, 5, and 10% critical values for the KPSS test with constant only are 0.739, 0.463 and 0.347, respectively. The 1, 5, and 10% critical values for the KPSS test with constant and trend are 0.126, 0.146 and 0.119, respectively.

Table 3 displays the BDS test statistics. The BDS test statistics were computed for various embedding dimensions including m = 2, 3, 4, 5, and 6. The distance value of $\varepsilon = 0.70$ was adopted for the test. The critical value of the BDS test at the 5 percent level of significance is ± 1.96. The null hypothesis that the residual of the time series is independent and identically distributed (i.i.d) is rejected if the computed test statistic exceeds the critical value (±1.96) at the 5 percent level of significance.

The test statistics reported in Table 3 overwhelmingly reject the null hypothesis that the REITs index return series are independently and identically distributed. The test statistics for the various dimensions are all greater than the critical value at the 1 percent level of significance. For example, for the dimension m =2, the test statistics 5.51, 4.81, and 5.17, respectively for all, equity and mortgage REITs index returns are statistically significant at the 1 percent level based on the reported *p*-values. The rejection of the null hypothesis implies that the REITs index return series are nonlinear which is suggestive of the existence of chaotic behavior. However, BenSaïda and Litimi (2013) suggest that the BDS test is not capable of differentiating chaotic from stochastic behavior. In addition to the BDS test, this study also applied a host of other nonlinearity tests including the Luukkonen, Reset1, Reset2, Tsay, Mcleod, White1 and White2. Details about these testing procedures can be found in Psaradakis and Spagnolo (2002) and Luukkonen et al. (1988) and as such will not be rehashed in this study given that they have been extensively applied in the literature.

Table 3: BDS Nonlinearity Test Results (January 1980 -December 2018)

	ALLR		**EQR**		**MOR**	
Dimension	*z-stat*	*p-value*	*z-stat*	*p-value*	*z-stat*	*p-value*
2	5.51^{***}	0.00	4.81^{***}	0.00	5.17^{***}	0.00
3	6.13^{***}	0.00	5.66^{***}	0.00	5.92^{***}	0.00
4	6.76^{***}	0.00	6.48^{***}	0.00	6.53^{***}	0.00
5	7.38^{***}	0.00	7.11^{***}	0.00	7.24^{***}	0.00
6	7.91^{***}	0.00	7.65^{***}	0.00	7.60^{***}	0.00

*** indicates level of significance at the 1% level. The null hypothesis is that the residuals are independent and identically distributed (iid). ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns

Table 3A: Luukkonen et al. (1988) Linearity Test Results (January 1980 -December 2018)

Series	**k**	**D**	**F-stat**	**p-value**
ALLR	2	2	4.87^{***}	0.00
EQR	2	3	6.68^{***}	0.00
MOR	2	2	2.43^{**}	0.03

*** and ** indicate rejection of the null hypothesis of linearity at the 1% and 5% level of significance, respectively. ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns

Table 3A presents the results from the Luukkonen, et al. (1988) linearity test. The appropriate lag lengths (*k*) and the delay parameters (*d*) were selected by Akaike Information Criterion. The results suggest that the null hypothesis of linearity should be rejected in all the cases. The test statistics 4.87, 6.68 and 2.43, respectively for all, equity and mortgage REITs index returns are statistically significant at least at the 5 percent level. These results corroborate those from the BDS tests in the sense that the REITs index return series were found to be nonlinear by both tests.

 ISSN 1927-033X

Table 3B: Linearity Test Results (January 1980 -December 2018)

Test	*ALLR*	*EQR*	*MOR*
RESET1	0.00***	0.00***	0.07*
RESET2	0.00***	0.00***	0.07*
TSAY	0.00***	0.00***	0.06*
MCLEOD	0.00***	0.00***	0.00***
WHITE1	0.00***	0.00***	0.00***
WHITE2	0.00***	0.00***	0.00***

*** and * indicate rejection of the null hypothesis of linearity at the 1% and 10% level of significance, respectively. ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns

Table 3B displays the results from the Reset1, Reset2, Tsay, Mcleod, White1 and White2 nonlinearity tests. Based on the reported p-values, the null hypothesis of linearity is rejected in favor of the alternative of nonlinearity. The various test results rejected the linearity hypothesis at least at the 1 percent level, except for mortgage REITs index returns where the null hypothesis is rejected at the 10% level of significance by the RESET1, RESET2 and TSAY procedures. However, Hsieh (1991) and Larrain (1991) point out that not all nonlinear dynamics exhibit a chaotic behavior. As such, these nonlinearity test results do not necessarily provide evidence as to whether the detected nonlinear structures are deterministic or stochastic. To this effect, the study applies the largest Lyapunov exponent (λ) test specifically designed to detect chaotic dynamics.

Table 4: 0−1 Test Results

Series	Test-statistic	Decision
Panel A: Full Sample Period (January 1980 -December 2018)		
ALLR	0.9976	Chaotic
EQR	0.9941	Chaotic
MOR	0.9980	Chaotic
Panel B: Pre-Crisis Sample Period (January 1980 to July 2007)		
ALLR	0.9977	Chaotic
EQR	0.9977	Chaotic
MOR	0.9987	Chaotic
Panel C: Post-Crisis Sample Period (January 2010 to December 2018)		
ALLR	0.9940	Chaotic
EQR	0.9938	Chaotic
MOR	0.9982	Chaotic

ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns

Having determined that all, equity, and mortgage REITs index returns are nonlinear, the study next utilizes the largest Lyapunov exponent procedure to determine whether the three series exhibit chaotic behavior. Prior to applying the largest Lyapunov exponent test, the study implements the 0-1 chaos test. Panel A of Table 4 displays the results from the 0-1 chaos test for the full sample period running from January 1980 through December 2018. As can be seen from Panel A of Table 4, the computed test statistics 0.99976, 0.9941, and 0.9980, respectively for all, equity and mortgage REITs index returns are very close to 1. These results reveal the presence of chaotic behavior in the three REITs index return series.

Panel B of Table 4 presents the 0-1 chaos test results for the pre-crisis period spanning January 1980 through July 2007. The reported test statistics 0.9977, 0.9977 and 0.9987, respectively for all, equity, and mortgage REITs index returns are very close 1, indicating the presence of chaotic dynamics in these series. Similarly, the results presented in Panel C of Table 4 for post-crisis period running from January 2010 through December 2018 show that the REIT return series are chaotic dynamics given that the computed statistics 0.9940, 0.9938 and 0.9982, respectively for all, equity and mortgage are close to 1. Taken together, the results from the 0-1 chaos tests suggest that the three REITs index return series exhibit chaotic behavior in all the sample periods. As such, the subprime crisis did not alter the structure of the REITs index return series. However, the results from the 0-1 test should be taken with caveat. Belaire-Franch (2018) points out that unlike the other chaos tests such as the largest Lyapunov exponent testing procedures, the 0-1 test does not provide a statistical framework and hence the researcher is left to determine whether the computed test is close to one. The subjective nature of the test makes it problematic. In addition, Hu, et al. (2005) suggest that the 0−1 test tends to misclassify both deterministic and weakly stochastic edge of chaos. They further argue that the test misclassifies strongly stochastic edge of chaos and weak chaos. Consequently, the 0−1 test is therefore not reliable in testing for chaos in financial time series.

Given the weaknesses associated with the 0−1 test, the study next implements the largest Lyapunov exponent. The largest Lyapunov exponent test has several attractive features over the 0-1 chaos technique. For instance, unlike the 0-1 chaos test, the Lyapunov exponent reveals the bifurcation points of the dynamical system. In addition, the Lyapunov exponent shows the super attractive points of the dynamical system. Panel A of Table 5 displays the results from the largest Lyapunov exponent for the full sample period running from January 1980 through December 2018. The test statistics were computed for the following embedding dimensions; 2, 3, 4, and 5 for robustness. As can be seen from Panel A of Table 5, the computed test statistics, 0.8309, 2.0106, 1.5253 and 2.3813 for all REITs index returns at the 2, 3, 4, and 5 embedding dimensions, respectively. The largest Lyapunov exponents for all REITs index returns are positive indicating that the null hypothesis that all REITs index returns exhibit chaotic behavior should not be rejected. Similar results are indicated for equity and mortgage REITs index returns for the full sample period. Taken together, the results presented in Panel A of Table 5 suggest that the three REITs index return series are chaotic dynamics for the full sample period. These results are not surprising given that the full sample includes the crisis period.

Panel B of Table 5 presents the test statistics from the largest Lyapunov exponent for the pre-subprime mortgage crisis period running from January 1980 through July 2007. Again, the estimated largest Lyapunov exponents for all, equity, and mortgage REITs index returns are positive. For example, the estimated largest Lyapunov exponents for all, equity, and mortgage REITs index returns are 0.2630, 2.5796 and 0.5402, respectively at dim = 5. Since the estimated largest Lyapunov exponents are positive, the null hypothesis of chaos is not rejected in all of the cases. These results indicate that the three REITs index return series exhibit chaotic behavior in the pre-crisis period.

 ISSN 1927-033X

Table 5: The Largest Lyapunov Exponent Test Results

Series	*Dim*	λ	*Dim*	λ	*Dim*	λ	*Dim*	λ	*Decision*
Panel A: Full Sample Period (January 1980 -December 2018) N=468									
ALLR	2	0.8309	3	2.0106	4	1.5253	5	2.3813	Chaotic
EQR	2	2.2340	3	2.5028	4	1.7675	5	3.8776	Chaotic
MOR	2	1.1462	3	2.1531	4	1.7536	5	1.5728	Chaotic
Panel B: Pre-Crisis Sample Period (January 1980 to July 2007) N=331									
ALLR	2	1.3915	3	0.5395	4	0.8284	5	0.2630	Chaotic
EQR	2	0.1895	3	2.1506	4	2.1344	5	2.5796	Chaotic
MOR	2	2.2866	3	1.7142	4	0.6244	5	0.5402	Chaotic
Panel C: Post-Crisis Sample Period (January 2010 to December 2018) N=108									
ALLR	2	-2.4020	3	-17.1522	4	-4.5865	5	-44.3064	Convergent
EQR	2	-0.8924	3	-14.3323	4	-2.4174	5	-63.3085	Convergent
MOR	2	-13.0700	3	-8.1170	4	-41.3559	5	-44.1651	Convergent

ALLR = All REITs index returns, EQR =Equity REITs index returns, and MOR= Mortgage REITs index returns. Dim=embedding dimension, λ= Largest Lyapunov exponent

Panel C of Table 5 presents the estimated largest Lyapunov exponent results for the post-subprime mortgage crisis period which runs from January 2010 to December 2018. The estimated largest Lyapunov exponents for all, equity, and mortgage REITs index returns are negative. For instance, the estimated Lyapunov exponents for all, equity, and mortgage REITs index returns are -4.5865, -2.4174 and -41.3559, respectively at dim=4. Based on these results, the null hypothesis of chaos in the REITs index return series is rejected. These results imply that REIT markets are nonchaotic following the subprime mortgage crisis period. This finding can be attributed to government intervention strategies which included the Housing and Economic Recovery Act of 2008, aimed at stabilizing the housing market.

5. Summary and Conclusions

This paper has examined the chaotic behavior of all REITs, equity REITs and mortgage REITs index returns for the time period running from January 1980 through December 2018 for the United States. To assess the effect of the subprime mortgage crisis, the sample was split into two — namely pre-crisis (January 1980 – July 2007) and post-crisis (January 2010 - December 2018). The empirical analysis commenced with the application of the ADF, Phillips-Perron and the KPSS unit root tests. The study next used a battery of procedures including the BDS, Reset1, Reset2, Tsay, Mcleod, White1, White2 and Luukkonen, et al. (1988) to test for nonlinearity in the three REITs index return series. To test for chaos, the study implemented the largest Lyapunov exponent. The results from the various unit root tests reveal that all, equity and mortgage REITs index returns are level stationary. The results from the nonlinearity tests suggest that the REITs index return series are nonlinear. The results from the largest Lyapunov exponent test indicate that all, equity, and mortgage REITs index returns exhibit chaotic behavior for the full sample and pre-subprime crisis periods. However, for the post subprime period, the study finds evidence that all, equity, and mortgage REITs index returns are nonlinear but not chaotic processes.

The results of this paper yield important implications for investors and portfolio managers.

 ISSN 1927-033X

First, the existence of nonlinearity suggests that the conventional linear models are not appropriate in modeling movements in REITs index returns. Accordingly, investors and portfolio managers should take nonlinearity in the structure of REITs index returns into consideration as they formulate strategies to maximize returns. Failure to do so, could in indeed weaken the benefits they derive from portfolio diversification in the long run. Second, the presence of chaos in the REITs index return series in the pre-subprime crisis period suggests that profitable opportunities may have existed for investors and portfolio managers to earn abnormal returns in the short run. However, the absence of chaotic dynamics in the structure of REITs index returns for the post subprime mortgage crisis period, implies that in the short run, speculators cannot exploit fluctuations in REIT markets, since the return series are nonlinear stochastic processes. Taken together, the results from this study indicate that although exploitable opportunity may have existed in the REIT markets prior to the subprime mortgage crisis, it nevertheless dissipated after the crisis due to the enactment of the Housing and Economic Recovery Act of 2008, which was intended to stabilize the housing market in the United States. The coronavirus (COVID-19) has adversely impacted the global economy and undoubtably the financial markets (Karabag 2020). REITs are not immune from the harmful effects of COVID-19. Consequently, future studies on REIT markets and their returns should adopt econometric models capable of accounting for shocks caused by COVID-19.

Acknowledgements

The author is grateful to two anonymous referees and the Chief Editor of JAEBR (Prof. Solmaz F. Karabag) for their insightful comments and suggestions that have improved the quality of the paper. The author is also thankful to Professor Kingsley Nwala for his constructive suggestions on the earlier version of the paper. The author is however responsible for all errors in the paper.

References

Ambrose, B. W., Ancel, E., Griffiths, M. 1992. The Fractal Structure of Real Estate Investment Trust Returns: The Search for Evidence of Market Segmentation and Nonlinear Dependency. *Journal of the American Real Estate and Urban Economics Association* 20:1, 25-54.

BenSaïda, A., Litimi, H. 2013. High Level Chaos in the Exchange and Index Markets. *Chaos Solitons Fractals* 54, 90-95.

Belaire-Franch, J. 2018. Exchange Rates Expectations and Chaotic Dynamics: A Replication Study. *Economics: The Open-Access, Open-Assessment E-Journal* 12, 1–8.

Bhattacharya, S.N., Bhattacharya, M., Roychoudhury, B. 2018. Behavior of the Foreign Exchange Rates of BRICS: Is it Chaotic? *The Journal of Prediction Markets* 11:2, 1-19.

Brock, W., Dechert, W., Scheinkman, J. 1996. A Test for Independence based on the Correlation Dimension. *Econometric Reviews* 15, 197–235.

Diaz, J. F. 2015. Chaos in Returns and Volatilities of the London Gold Fix. *Chaos and Complexity Letters.* 9:3, 205-219.

Dickey, D., Fuller, W. 1979. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association* 74:366, 427–431.

Hsieh, D. 1991. Chaos and Nonlinear Dynamics: Application to Financial Markets. *The Journal of Finance*. 46, 1839-1877.

Hu, J., Tung, W.-w., Gao, J., Cao, Y. 2005. Reliability of the 0 – 1 Test for Chaos. *Physical Review* 72, 056207.

Jirasakuldech, B., Emekter, R. 2012. Nonlinear Dynamics and Chaos Behaviors in the REIT Industry: A Pre- and Post-1993 Comparison. *Journal of Real Estate Portfolio Management* 18:1, 57-77.

Kantz, H. 1994. A Robust Method to Estimate the Maximal Lyapunov Exponent of a Time Series. *Physics Letters A* 185:1, 77–87.

Karabag, S. F. 2020. An unprecedented global crisis! the global, regional, national, political, economic and commercial impact of the coronavirus pandemic. *Journal of Applied Economics and Business Research* 10, 1-6.

Kumar, A.S., Kamaiah, B. 2016. Efficiency, Non-linearity, and Chaos: Evidence from BRICS Foreign Exchange Markets. *Theoretical and Applied Economics*, 23:1, 103-118.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., Shin, Y. 1992. Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root. *Journal of Econometrics* 54, 159--178.

Larrain, M. 1991. Testing Chaos and Nonlinearities in T-Bill Rates. *Financial Analysts Journal*. 47:5, 51–62.

Liow, K. H., Webb, J. R. 2008. Nonlinear Return Dependence in Major Real Estate Markets Nonlinear Return Dependence in Major Real Estate Markets. *Journal of Property Research*, 25:4, 285–319.

Litimi, H., BenSaïda, A., Belkacem, L., Abdallah, O. 2019. Chaotic Behavior in Financial Market Volatility. *Journal of Risk*, 21:3, 27-53.

Luukkonen, R. Saikkonen P. and Terasvirta, T. 1988. Testing Linearity against Smooth Transition Autoregressive Models. *Biometrika* 75, 491-500.

McQueen, G., Thorley, S. 1991. Are Stock Returns Predictable? A Test Using Markov Chain. *Journal of Finance* 16:1, 239–263.

Mishra, R. K., Sehgal, S, Bhanumurthy, N.R. 2011. A Search for Long-Range Dependence and Chaotic Structure in Indian Stock Market. *Review of Financial Economics* 20, 96–104.

Okunev, J., Wilson, P.J., Zurbruegg, R. 2000. The Causal Relationship between Real Estate and Stock Markets. *Journal of Real Estate Finance and Economics*. 21:3, 251–262.

Phillips, P., Perron, P. 1989. Testing for a Unit Root in Time Series Regression. *Biometrica* 75, 335–346.

Psaradakis, Z., Spagnolo, N. 2002. Power Properties of Nonlinearity Tests for Time Series with Markov Regimes. *Studies in Nonlinear Dynamics and Econometrics*, 6:3, 1-14.

Ramsey, J.B., Rothman, P. 1996. Time Reversibility and Business Cycle Asymmetry. *Journal of Money, Credit, and Banking*, 28, 1–21.

Rosenstein, M.T, Collins, J.J., De Luca, C. J. 1995. A practical Method for Calculating Largest Lyapunov Exponents from Small Data Sets. *Physica D* 65, 117–134.

Serletis, A., Shintani, M. 2003. No Evidence of Chaos but Some Evidence of Dependence in the U.S. Stock Market. *Chaos, Solitons and Fractals* 17, 449-454.

Tiwari, A. K., Gupta, R. 2019. Chaos in G7 Stock Markets Using Over One Century of Data: A Note. *Research in International Business and Finance* 47, 304-310.

Webel, K. 2012. Chaos in German Stock Returns - New Evidence from the 0–1 Test. *Economics Letters* 115:3, 487 – 489.

Wolf, A., Swift, J. B., Swinney, H. L., Vastano, J. A. 1985. Determining Lyapunov Exponents from a Time Series. *Physica D Nonlinear Phenomena* 16:3, 285–317.

 ISSN 1927-033X