# Setting a Uniform Price vs. Discriminatory Prices by a Monopolist 

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#### Abstract

This paper uses both a geometrical and mathematical analysis to explain monopolistic third-degree price discrimination, and it also shows how price discrimination affects society. A frequent policy question in the literature on price discrimination is whether to allow third-degree price discrimination or enforce uniform pricing. A key feature to understanding this issue in the context of imperfectly competitive markets is the impact of price discrimination on output. The article shows that a monopoly facing downward-sloping linear demands and constant marginal costs will obtain higher profits under price discrimination than under a single-price strategy, but price discrimination lowers welfare if the total output does not change. When price discrimination causes total output to increase, then this practice will have a beneficial effect on overall welfare.


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## 1. Introduction

This paper focuses on third-degree price discrimination, a material that can supplement any textbook chapter on monopoly. Third-degree price discrimination is a price strategy that involves a firm charging different market segments different prices for the same product. Price differences cannot be explained by the difference in the marginal cost of making the goods available to different buyers. A monopolistic producer can charge different prices to consumers based on their willingness to pay. A consumer's willingness to pay is measured by the price elasticity of demand. The firm will charge a higher price to consumers with less elastic demand. Although, in principle, price discrimination is possible for any company whose customers are willing to pay different prices for a good when the firm knows that there are different segments of the market, typically this practice is associated in microeconomics and industrial organization with producers that operate as monopolies in a market. Examples include discount rates on the train for senior citizens, student discounts on museum tickets, and so on.

Price discrimination is an important and classic topic in any microeconomics course. Students learn that by charging a different price in different market segments, an unregulated monopolist can earn higher profits than by charging the same price to all buyers. However, most of the textbooks explain price discrimination using mainly graphs, and they give only intuitive ideas about this topic (e.g., Varian, 2003). But students should go beyond graphs to understand the behavior of the monopolies. This paper uses both a geometrical and mathematical analysis to explain monopolistic third-degree price discrimination. The article demonstrates that a monopoly facing

[^0]downward-sloping linear demands and constant marginal costs will obtain higher profits under price discrimination than under a single-price strategy.

Most microeconomics textbooks also explain how monopolies determine output and price and why unregulated monopolies have undesirable outcomes: they produce lower output and charge a higher price than perfect competitors. This issue is important since students must understand that because a monopoly produces an inefficient level of production, governments often try to regulate it or break it up into several firms. However, microeconomics manuals usually illustrate the aggregate welfare loss due to monopoly power using only graphs. To rectify this neglect, we compute the deadweight welfare loss of monopoly, an estimate of the social cost of monopoly. In applied work, calculations are focused mainly on the U.S. economy. The initial efforts were due to Harberger (1954), who found the deadweight loss from a monopoly in the manufacturing industry to be equal to (at most) 0.1 percent of GNP. Later, Posner (1975) estimated the total social costs of monopolies at about 1.8 percent of GNP, and Parker and Connor (1979) evaluated the consumer loss due to a monopoly in the food-manufacturing industries in the $\$ 12$ to $\$ 14$ billion range. More recently, welfare losses due to imperfect competition can be found in Ritz (2014).

The paper also highlights the importance of our analysis for a variety of questions related to public regulation, such as public policies toward price discrimination. It is a fact that price discrimination practiced by a monopoly has effects on social welfare. Academic economists are interested in knowing under what circumstances price discrimination harms welfare, and whether the prohibition of price discrimination could be justified, and under what other circumstances price discrimination has positive effects on welfare. Although the relevant economic literature shows that the welfare implications of price discrimination are ambiguous, the more that price discrimination results in increased output or indeed opens up new markets, the more probable it is to have a positive impact on economic welfare (Swedish Competition Authority, 2005). Research on these issues is very useful not only for governments and competition authorities but also for competition law experts, mainly in industries such as telecommunications, water and gas delivery, or public transportation. The previous ideas may give a first criterion that could be used to evaluate the anti-competitive effects of price discrimination: to favor consumers, price discrimination should at least increase the total quantity. If discrimination does not increase total amounts, it is certainly bad from a welfare point of view. Thus, this paper seeks to understand the conditions under which price discrimination leads to an increase in aggregate output relative to uniform pricing. The first lesson is that production must increase more in the weak market than it diminishes in the strong one since each unit is more valued in the latter. The second lesson is that if discrimination banning leads the monopoly firm to leave the segment of consumers with low valuation (weak market) in order to serve only high-value consumers (strong market) at a higher price, then discrimination banning is a bad thing. It leads, in this case, to a reduction in total quantity, which is certainly disadvantageous for consumers.

The paper comprises six sections, besides this introduction. The second section presents a literature review. We briefly review the known results on the welfare effects of third-degree price discrimination. A necessary, but not sufficient, condition for price discrimination to increase static Marshallian welfare-defined as the sum of consumer and producer surpluses-is that total output should increase. Section 3 provides a practical example to help readers understand monopoly pricing behavior: how a monopoly determines the quantity to produce and the price to charge, and why the monopolist tries to charge different prices to different customers. We consider a monopolist that faces two separate linear demand curves with the same constant marginal cost in each sub-market. Next, the article proves that welfare may be higher with third-degree price
discrimination than with a non-discriminating monopoly if the output is greater with discrimination. A necessary condition for welfare to rise is that total production must increase. Price discrimination causes the strong market's output reduction to be less than the weak market's output increase. The penultimate section presents two analyses. A first case study is proposed to prove that there are situations in which a monopolist that does not discriminate serves only one sub-market because it gets a greater profit. But allowing price discrimination would encourage the firm to open a second sub-market, with an increase in welfare (consumer surplus and profit would increase in the second group). The welfare of the first group would not change. Case 2 shows that a monopolist that faces different linear demand curves in two sub-markets with the same constant marginal cost in each sub-market sells the same quantity without and with price discrimination, and price discrimination lowers total welfare. In this case, it might be warranted to ban price discrimination. The article finishes with a section of conclusions.

## 2. Literature Review

The examination of price discrimination has deep roots in economics. According to Ekelund (1970), the theory of price discrimination and the study of product differentiation were already present in the works of Jules Dupuit. He was a French economist-engineer of the 19th century who created a formal analytical frame within which to discuss the effects of discrimination upon prices, output, and economic welfare, although the scientific treatment was set later by Arthur Pigou and Joan Robinson. In The Economics of Welfare, Pigou (1920) identified three categories of price discrimination and assessed their effects: first-degree, second-degree, and third-degree price discrimination. First-degree price discrimination involves charging every consumer the maximum amount that s/he is willing to pay for each unit of the economic good (this removes all consumer surplus). Second-degree price discrimination is the practice of setting two or more prices for a good depending on the amount purchased. Third-degree price discrimination is the practice of charging different prices to different consumers for the same good. In third-degree price discrimination, a producer identifies separable market segments, each of which has its own demand for its product. The firm then sets a price for each segment in accordance with that segment's demand elasticity.

The typical analysis of third-degree price discrimination involves action by a monopolist to increase profits by dividing the market so that each class of consumers pays a price closer to the buyers' reservation prices-the maximum price the buyer is willing to pay (Gifford \& Kudrle, 2010). In this regard, Carroll and Coates (1999) identified three necessary market conditions for companies that wish to employ price discrimination. First, the consumers should experience heterogeneous utilities from the good, and therefore they should have different price elasticities of demand. Through price discrimination, producers can extract consumer surplus and hence raise their profits. When customers have different valuations for the product or when there are different groups of customers with identifiable sensitivity to prices (e.g., price elasticity), price discrimination allows the firm to exploit these differences to increase profits. The degree to which producers can extract consumer surplus will depend on the information available on consumer preferences. The extreme case is first-degree price discrimination, in which the company knows the preferences of each customer and can extract the entire consumer surplus. The finer the information, the finer the pricing strategies that can be implemented, and the larger the scope for extracting consumer surplus. In many situations, it is intuitive that when a firm is allowed to engage in price discrimination, some of its prices will fall while others will rise. That is to say, the nondiscriminatory price is some "average" of the discriminatory prices. Second, the firm must have some market power. The term "market power" is central to considerations about price discrimination. Lerner (1934) offered the first measurement of monopoly power which focused attention on the shared characteristic of producers in imperfect competition: monopoly power is
some power over price. Lerner (1934) employed the term monopoly power, to refer to all situations in which price exceeds marginal cost. Third, the firm must be able to control the sale of its products; the producer must separate customers into distinct markets and prevent the reselling of the product from customers in one market to clients in another market.

The treatment of price discrimination raises rather problematic topics, and the economic analysis of the effects of price discrimination does not offer general and straightforward conclusions (Ikeda \& Toshimitsu, 2010). Since Pigou's (1920) seminal work, the central question in the analysis of third-degree price discrimination has been about its welfare effects. The traditional study of the welfare effects of price discrimination focuses on the impact of discrimination on total output. The influence on social welfare of third-degree price discrimination was first examined by Robinson (1933). In particular, price discrimination necessarily decreases social welfare if demands are linear because aggregate output remains constant. Schmalensee (1981) reexamined this question and presented several new results. In particular, he noted that a necessary condition for price discrimination to increase social welfare-defined as consumer surplus plus producer surplus-is that output must increase.

An analysis of the determinants of the course of output when several markets with nonlinear demand curves can be served both before and after discrimination is Robinson's unique contribution to the pure theory of discrimination, and neither Dupuit nor Pigou (Ekelund, 1970). Robinson (1933) concluded that when a monopolist price discriminates, whether aggregate output increases depends on the relative curvature of the segmented demand curves. Specifically, in the case of two segments, if the "adjusted concavity" of the more elastic market is greater than the adjusted concavity of the less elastic market at a uniform price, then output increases with price discrimination; when the reverse is true, aggregate output decreases. In her study of third-degree price discrimination under monopoly, Robinson (1933) characterized a monopolist's two markets as "strong" and "weak." By definition, a price-discriminating monopolist sets the higher price in the strong market and the lower price in the weak market. When a market segment has linear demand, the adjusted concavity is zero. It follows that when demand curves are linear-providing all markets are served-price discrimination has no effect on aggregate output (Stole, 2007). As we said before, Schmalensee (1981) showed that if demands for the products are independent and marginal costs are constant, then if welfare increases with discrimination, the total output must increase. Varian (1985) extended this argument as well to allow for cross-price effects.

Although using a simple model of third-degree price discrimination, assuming two independent linear demands, Kwon (2006) derived the probability that price discrimination improves social welfare, price discrimination has a negative reallocation effect. This can only be overcome if the quantity effect is positive, that is, if price discrimination induces a higher output. Aguirre et al. (2010) found sufficient conditions-based on the curvatures of direct and inverse demand functions-for third-degree price discrimination to increase (or decrease) social welfare. The main results showed that the output effect is stronger than the misallocation effect. Price discrimination raises social welfare when inverse demand in the weak market is more convex than that in the strong market and the price difference with discrimination is small, and discrimination decreases welfare when the direct demand function is more convex in the high-price market. Later, Cowan (2012) showed that total consumer surplus is higher with discrimination if the ratio of passthrough to the price elasticity (at the uniform price) is the same or larger in the weak market. As an application, the paper shows that discrimination always rises surplus for logit demand functions whose pass-through rates exceed 0.5 (thus demand is convex). The author notes that an increase in consumer surplus ensures an increase in social welfare, given that price discrimination increases profits. Consequently, with this demand family, results are different from those under linear
demand: the output effect always dominates the misallocation effect for logit demand functions, with pass-through rates exceeding 0.5 .

The results above assume that all markets have positive demand under both price discrimination and uniform pricing. This supposition may fail to hold. The uniform pricing firm may earn its highest profit at a price that excludes demand from low-reservation-price users. When new markets may open, then price discrimination can lead to Pareto-welfare improvements (Hausman \& Mackie-Mason, 1988). Further, and quite important for a new product, declining marginal costs from scale and learning economies may be possible with increasing output (Hausman \& Mackie-Mason, 1988).

We can conclude this section by saying that a monopolist is almost always better off when it can price discriminate, but a recurring policy question in the literature on price discrimination is whether to allow third-degree price discrimination or enforce uniform pricing. From the viewpoint of society, it is impossible to say a priori whether price discrimination is desirable or not. The welfare and efficiency consequences of price discrimination by a monopoly in comparison to simple uniform monopoly pricing are ambiguous (Schmalensee, 1981). Price discrimination could increase or reduce efficiency compared to uniform pricing depending on the shapes of the underlying demands for the services as well as the attributes of the firm's cost function. A key ingredient to understanding this question in the context of imperfectly competitive markets is the impact of price discrimination on output. Ideally, then, policy towards price discrimination should be founded on a good economic understanding of the market in question. Since it is impractical to require competition bodies to have a good economic understanding of all markets, some broad rules of thumb are needed. If all markets are served with uniform pricing, demand functions are linear, and the marginal cost is constant, then total welfare is lower with price discrimination than with uniform pricing. This is because total production is the same in both cases. Therefore, an increase in aggregate output is a necessary condition for price discrimination to increase welfare. Pigou (1920) and Robinson (1933) gave early proof of this result. Schmalensee (1981), Varian (1985), Schwartz (1990), Layson (1998), Ikeda and Nariu (2009), and Bergemann et al. (2015), among others, have also analyzed these welfare effects with varying degrees of generality.

## 3. Materials and Methods

Let's consider a monopoly that sells a good in two sub-markets, 1 and 2 , which have independent consumer demands. The firm faces the following downward-sloping linear demands for its product

$$
\begin{align*}
& p_{1}=300-q_{1}  \tag{1}\\
& p_{2}=180-q_{2}
\end{align*}
$$

If the cost function is given by $\mathrm{TC}=40 Q$, should the monopoly charge a uniform price or discriminate? We suppose that the consumers in each sub-market are not able to resell the good.

### 3.1 A Non-Discriminating Monopolist

If the demands for the good in each sub-market are independent, the market demand is equal to the sum of the individual demand functions for the good. But when we aggregate the demand for a good over several independent groups of sub-markets, we must be cautious to take into account the fact that the quantity demanded may be nil in one group. It is particularly the case for linear demand functions. When we aggregate independent demand curves to build a market demand
curve, the latter will always have at least one kink if the demand curves of each sub-market are linear in price with different "reservation prices."

Let us first express the direct demand functions as a function of only one price, $p$

$$
\begin{align*}
& q_{1}=300-p \\
& q_{2}=180-p \tag{2}
\end{align*}
$$

The demand for the market is given by $Q(p)=q_{1}(p)+q_{2}(p)$. We cannot just add the two demand functions together, though, because the type 2 demand will be negative for certain prices that create positive demand from type 1 customers. Thus, the market demand is given by (3)

$$
\left\{\begin{array}{lr}
Q=300-p \quad \text { if } 180 \leq p \leq 300 \\
Q=480-2 p & \text { if } 0 \leq p<180 \tag{3}
\end{array}\right.
$$

The direct demand function for the market (3) characterizes the quantity of the good that the firm can expect to sell in the market if it charges any particular price $p$.

We now have the inverse demand function for the market (4). We note that the total demand curve has a kink at $Q=120$

$$
\left\{\begin{array}{rr}
p=300-Q & \text { if } \quad 0 \leq Q \leq 120  \tag{4}\\
p=240-\frac{Q}{2} & \text { if } Q>120
\end{array}\right.
$$

Total revenue (price • quantity) is given by (5)

$$
\left\{\begin{array}{lr}
T R=300 Q-Q^{2} & \text { if } \quad 0 \leq Q \leq 120  \tag{5}\\
T R=240 Q-\frac{Q^{2}}{2} & \text { if } Q>120
\end{array}\right.
$$

The marginal revenue equations are given by $\frac{d T R}{d Q}$

$$
\left\{\begin{array}{lr}
M R=300-2 Q & \text { if } \quad 0 \leq Q \leq 120  \tag{6}\\
M R=240-Q & \text { if } Q>120
\end{array}\right.
$$

The monopolist, like any other firm, sets $\mathrm{MR}=\mathrm{MC}$, marginal revenue equals marginal cost $(=\$ 40)$, to determine the optimal quantity to produce $\left(Q^{*}=200\right)$

$$
\begin{align*}
& 40=300-2 Q ; Q=130(\text { out of the interval })  \tag{7}\\
& 40=240-Q ; Q^{*}=200 \text { units }(\text { solution })
\end{align*}
$$

Now, we compute the price charged by a single-price monopolist: $p=240-$ 200/2; $p^{*}=\$ 140$.

In Figure 1, the profit-maximizing $Q^{*}$ is found at the intersection of the marginal cost curve MC and the marginal revenue curve $\mathrm{MR}_{\mathrm{M}}$. A profit-maximizing monopolist would produce an output of 200 units; the equilibrium E occurs in the area of demand where both sub-markets are served. Given the optimal total production $Q^{*}$, the monopolist chooses the price, which is the height
of the demand curve. The uniform price $p^{*}{ }_{\mathrm{m}}$ is the profit-maximizing price when all markets are charged the same price. A single price of $\$ 140$ is charged to all consumers.

Finally, we compute the firm's profit $\pi$ (total revenue minus total costs)

$$
\begin{equation*}
\pi=(\text { price } \cdot \text { quantity })-\text { total costs } \tag{8}
\end{equation*}
$$

Replacing the values for $p$ and $Q$ into the profit equation, we find

$$
\begin{gather*}
\pi=140(200)-40(200)=28000-8000  \tag{9}\\
=\$ 20,000
\end{gather*}
$$

### 3.2 A Two-Price Discriminating Monopolist

Figure 1 also shows third-degree price discrimination. With price discrimination, the market is divided into two segments (groups or sub-markets). The curves labeled $D_{1}$ and $M R_{1}$ represent the demand and marginal revenue, respectively, for group 1 consumers. This demand curve is relatively vertical; it's less elastic. Likewise, the curves labeled $\mathrm{D}_{2}$ and $\mathrm{MR}_{2}$ represent the demand and marginal revenue, respectively, for group 2 consumers. Group 2's flatter demand curve indicates a more elastic demand. The marginal cost is constant and labeled MC. The profitmaximizing quantity for each sub-market corresponds to the output where the group's marginal revenue equals marginal cost. Since MC must equal $\mathrm{MR}_{1}$ and $\mathrm{MR}_{2}$, we can draw a horizontal line leftwards from the intersection in E to find the profit-maximizing quantities $q_{1}^{*}$ and $q_{2}^{*}$ (Weber \& Pasche, 2008; Salas-Velasco, 2021). From the demand curve in each sub-market, we can determine the profit-maximizing prices $p_{1}^{*}$ and $p_{2}^{*}$ as well. Hence, the sub-market with the more elastic demand-the market that is more price sensitive-is charged the lower price.


Figure 1. Monopoly Power and Firm Pricing Decisions

Figure 1 shows a geometrical analysis of a uniform price vs. third-degree price discrimination by a profit-maximizing monopoly. Heterogeneity between buyers comes from their willingness to pay; group 1 has a higher willingness to pay than group 2. If the firm is allowed to discriminate, it will charge two different prices, $p_{1}^{*}$ and $p_{2}^{*}$, to consumers characterized by a high and low willingness to pay, respectively. When the firm is not allowed to discriminate (that is, under a uniform price regime), it will charge a single price of $p_{\mathrm{m}}^{*}$ to both classes of consumers. Total output remains unchanged during this process ( $=200$ units). Joan Robinson (1933) already demonstrated that a monopolist's output remains constant with linear demand curves, whether or not the monopolist discriminates.

The pricing decision of the discriminating monopoly is demonstrated in Table 1. Under thirddegree price discrimination, the firm can charge different prices to consumers belonging to different groups or sub-markets ( 1 and 2 ). The inverse demand function for type 1 buyers is $p_{1}=$ $300-q_{1}$ with a corresponding marginal revenue of $\mathrm{MR}_{1}=300-2 q_{1}$. The inverse demand function for type 2 buyers is $p_{2}=180-q_{2}$, generating a marginal revenue of $\mathrm{MR}_{2}=180-2 q_{2}$. The monopolist chooses quantities in each consumer group such that the marginal revenue in each submarket is equal to the marginal cost: $\mathrm{MR}_{1}=\mathrm{MR}_{2}=\mathrm{MC}$. Equating the marginal revenue with the marginal cost of $\$ 40$, we can solve for $q_{1}^{*}=130$ and $q_{2}^{*}=70$. The price charged for type 1 consumers is $p_{1}^{*}=170$, and the corresponding price for type 2 customers is $p_{2}^{*}=110$.

Moving from non-discrimination to price discrimination raises the firm's profits. The monopolist's profit function is now given by

$$
\begin{equation*}
\pi^{d}=170(130)+110(70)-40(200)=29800-8000=\$ 21,800 \tag{10}
\end{equation*}
$$

Table 1. Two Different Segments, Two Different Prices: How Does the Monopolist Determine the Optimal Prices to Charge in Each Sub-Market?

Sub-market 1

$$
\begin{gathered}
T R_{1}=300 q_{1}-q_{1}^{2} ; M R_{1}=300-2 q_{1} \\
40=300-2 q_{1} ; q_{1}^{*}=130 \text { units } \\
p_{1}=300-q_{1} ; p_{1}^{*}=\$ 170
\end{gathered}
$$

Sub-market 2

$$
\begin{gathered}
T R_{2}=180 q_{2}-q_{2}^{2} ; M R_{2}=180-2 q_{2} \\
40=180-2 q_{2} ; q_{2}^{*}=70 \text { units } \\
p_{2}=180-q_{2} ; p_{2}^{*}=\$ 110
\end{gathered}
$$

Table 1 shows the behavior of a monopoly that is practicing third-degree price discrimination. The total output $Q^{*}$ of 200 units must be divided between the two sub-markets so that MC is equal to MR in each segment. Thus, in sub-market 1,130 units will be sold at $\$ 170$ each, and in sub-market 2,70 units will be sold at $\$ 110$ each. Discriminatory prices $p_{1}^{*}$ and $p_{2}^{*}$ are profit-maximizing prices.

The monopolist's profit under third-degree price discrimination is greater than the nondiscriminating monopolist's profit. Total output remains unchanged during this process ( $=200$ units), and total costs do not change. The profit gain from price discrimination $(=\$ 1,800)$ is, in fact, the additional revenue from increased sales to group 2 minus the revenue forgone from reduced sales to group 1-total revenue increases thanks to the reallocation of 30 units. The output in sub-market 1 decreases by 30 units under price discrimination (from 160 to 130 units), while the output in sub-market 2 increases by 30 units (from 40 to 70 units).

$$
\begin{align*}
\pi=\{140 \cdot 160+ & 140 \cdot 40\}-40(200)=(22400+5600)-8000=28000-8000 \\
& =\$ 20,000 \\
\pi^{d}=\{170 \cdot 130 & +110 \cdot 70\}-40(200)=(22100+7700)-8000=29800-8000  \tag{11}\\
& =\$ 21,800
\end{align*}
$$

### 3.3 Inverse Elasticity Rule for Third-Degree Price Discrimination

The classic theory of third-degree price discrimination by a monopolist selling to several separate markets is straightforward: a higher price $\left(p_{1}\right)$ is charged to the low elasticity group, and a lower price $\left(p_{2}\right)$ is charged to the high elasticity group. This result is intuitive: consumers with
less elastic demand are less price-sensitive, and it is optimal for the monopolist to charge them more. This result can be shown rigorously by expressing the profit-maximizing condition in the following formula at the optimum (here $\varepsilon$ is the absolute value of the price elasticity of demand)

$$
\begin{equation*}
p_{1}\left(1-\frac{1}{\varepsilon_{1}}\right)=p_{2}\left(1-\frac{1}{\varepsilon_{2}}\right) ; \text { or } \quad \frac{p_{1}}{p_{2}}=\frac{1-\frac{1}{\varepsilon_{2}}}{1-\frac{1}{\varepsilon_{1}}} \tag{12}
\end{equation*}
$$

where sub-market 1 has a price elasticity of demand of $\varepsilon_{1}$ and sub-market 2 of $\varepsilon_{2}$, respectively. It is very beneficial for the price discriminator to determine the optimum prices in each market segment by using this formula. If the demand in sub-market 2 is more elastic than in sub-market 1 , then the price will be lower in sub-market 2 .

Table 2 shows algebraically that the sub-market with the lower price elasticity of demand will be charged the higher price. The discriminating monopolist would charge a high price in submarket 1 (the strong market) and a low price in sub-market 2 (the weak market). Consumers in submarket 1 have a high willingness to pay, and consumers in sub-market 2 have a low willingness to pay. In general, the implication of this is that if firms can practice third-degree price discrimination across markets, they should charge higher prices in markets where elasticity is low and lower prices in markets with high elasticities. If all consumers valued the good equally, there would be no rationale for the firm to offer different prices to different customers. The extent to which the firm has accurate information about the differences in consumer preferences determines the degree of price discrimination that the firm can employ.

Table 2. Optimal Third-Degree Price Discrimination

| Sub-market 1 |  |
| :---: | :---: |
| $q_{1}=300-p_{1}$ | Sub-market 2 |
| $q_{1}=130 ; p_{1}=170$ | $q_{2}=180-p_{2}$ |
| $\varepsilon=-\frac{d q}{d p} \frac{p}{q}=-(-1) \frac{170}{130}=\frac{17}{13}(=1.31)$ | $q_{2}=70 ; p_{2}=110$ |
| $\varepsilon=-\frac{d q}{d p} \frac{p}{q}=-(-1) \frac{110}{70}=\frac{11}{7}(=1.57)$ |  |
| $170\left(1-\frac{13}{17}\right) \equiv 110\left(1-\frac{7}{11}\right) \equiv M C$ of $\$ 40.00$ |  |
| Since $\varepsilon_{1}<\varepsilon_{2}$, at the optimum $p_{1}^{*}>p_{2}^{*}$ |  |

A discriminatory pricing monopoly charges consumers in different groups different unit prices. Some buyers in the market will almost certainly be prepared to pay a higher price for a product than other buyers. Table 2 shows that if there were only two separate sub-markets, the higher price would be charged in the segment with the less elastic demand curve. By charging a different price in the two segments, the monopolist can earn higher profits than it would if it charged the same price of $\$ 140$.

## 4. The Welfare Economics of Monopoly

### 4.1 The Social Cost of Monopoly Without Price Discrimination

A perfectly competitive industry operates at a point where price equals marginal cost, while an industry under monopoly operates where the price is greater than marginal cost, as depicted in Figure 2. In general, the price will be higher and the quantity lower if a firm behaves monopolistically rather than competitively. For this reason, buyers will typically be worse off in an industry organized as a monopoly than in one organized competitively.

In the following lines, much is made of the relationship between aggregate output and welfare. Because the monopolist operates at an inefficient level of production, there is a deadweight loss, which represents a true decrease in welfare. Economic well-being can be measured as the sum of consumer surplus and producer surplus. It measures the excess value generated for all participants in this market by market activity. Deadweight loss occurs in the economy when total welfare is not maximized.

There is a straightforward equation for the deadweight loss due to monopoly (DWL) or welfare loss (triangle in Figure 2)

$$
\begin{equation*}
D W L=\frac{\left(p_{m}-M C\right)\left(Q_{p c}-Q_{m}\right)}{2} \tag{13}
\end{equation*}
$$

In Table 3, we compute the social cost of the non-discriminating monopoly used in our previous example. Deadweight loss can be quantified as the loss of total welfare due to monopoly pricing, which is equal to $\$ 10,000$.

Table 3. Deadweight Loss Due to Monopoly

$$
\begin{gathered}
p=M C ; 240-\frac{Q}{2}=40 ; 200=\frac{Q}{2} ; Q_{p c}=400 \text { units } \\
D W L=\frac{(140-40)(400-200)}{2}=\$ 10,000
\end{gathered}
$$

Table 3 shows the estimation of the deadweight loss due to monopoly. Assessing the welfare loss due to monopoly pricing is essentially an estimate of the triangle AEM in Figure 2 (the shaded triangular area displays the inefficiency of monopoly). We first consider a competitive market as a benchmark for maximum social welfare.


Figure 2. A Way to Measure the Total Loss in Efficiency Due to a Monopoly

Figure 2 shows that firms in a perfectly competitive market will produce at point E , the intersection of the market demand and marginal cost curves. However, the monopolist will produce at point M , the intersection of marginal cost and marginal revenue curves. In a competitive market, therefore, the price will be lower $\left(p_{\text {pc }}\right)$ and the quantity produced greater ( $Q_{\mathrm{pc}}$ ) than under a monopoly. From an economic point of view, there is an efficiency loss caused by going from perfect competition to monopoly. The deadweight welfare loss due to monopoly is the triangle AEM, and the rectangle $p_{\mathrm{m}} \mathrm{AM} p_{\mathrm{pc}}$ describes the redistribution from consumers to the monopolist. The deadweight loss increases as the demand curve becomes less elastic at monopoly equilibrium (Carlton \& Perloff, 2005).

### 4.2 Third-Degree Price Discrimination and the Welfare Cost of Monopoly

What are the welfare effects when a monopolist practices third-degree price discrimination rather than setting a uniform price in all sub-markets? Even in the simplest two-market case of linear demand, one should be able to determine when price discrimination is likely to be welfare augmenting or decreasing. When demand curves are linear and the marginal cost is constant, providing all markets are served, price discrimination has no effect on aggregate output, and it will cause welfare to fall. In our example, the welfare loss from price discrimination could be calculated as follows

$$
\begin{align*}
& D W L_{1}=\frac{(170-40)(260-130)}{2}=\$ 8,450  \tag{14}\\
& D W L_{2}=\frac{(110-40)(140-70)}{2}=\$ 2,450
\end{align*}
$$

In (14), 260 and 140 would be the quantities produced if each sub-market, respectively, if they were perfectly competitive ( price $=$ marginal cost).

As an alternative way to do this, we can use the formula for calculating a triangle suggested in Hotelling (1938) and that we borrow from Rhoades (1982)

$$
\begin{equation*}
W_{i}=\frac{1}{2} \pi_{m i}^{2} \frac{1}{T R_{i}} \varepsilon_{i} \tag{15}
\end{equation*}
$$

where: $W_{i}=$ welfare loss in the ith market; $\pi_{m i}=$ monopoly profits in the ith market; $\mathrm{TR}_{\mathrm{i}}=$ total revenue in the ith market; and $\varepsilon_{i}=$ price elasticity of demand in the ith market. Thus

$$
\begin{align*}
& W_{1}=\frac{1}{2}(170 \cdot 130-40 \cdot 130)^{2} \frac{1}{(170 \cdot 130)} \frac{17}{13}=8450 \\
& W_{2}=\frac{1}{2}(110 \cdot 70-40 \cdot 70)^{2} \frac{1}{(110 \cdot 70)} \frac{11}{7}=2450 \tag{16}
\end{align*}
$$

The total welfare loss under price discrimination is $\$ 10,900$. Thus, monopolistic third-degree price discrimination increases welfare loss by $\$ 900$. It means that social welfare is even lower under third-degree price discrimination than under single-price monopoly market conditions. That is the case when the output does not change under third-degree price discrimination.

Assuming profit maximization and linear demands, the relationship between the profit gain from price discrimination $(\Delta \pi)$ and the welfare loss change ( $\Delta \mathrm{DWL}$ ) is given by (Cowling \& Mueller, 1981)

$$
\begin{align*}
& \triangle D W L=\frac{1}{2} \Delta \pi \\
& 900 \equiv \frac{1}{2}(1,800) \tag{17}
\end{align*}
$$

The welfare loss change comes from the transfer of units of product from group 1, which values the good more highly, to group 2, which values the product less highly. Consumers in submarket 1 put a very high value on the good, and they are better off under single-price monopoly (consumer surplus $=\$ 12,800$ ) than under price discrimination (consumer surplus $=\$ 8,450$ ); there is a reduction of $\$ 4,350$ in consumer surplus. In contrast, consumers with lower reservation prices (sub-market 2) purchase 40 units from the single-price monopoly (consumer surplus $=\$ 800$ ) and 70 units from the price-discriminating monopoly (consumer surplus $=\$ 2,450$ ); there is an increase of $\$ 1,650$ in consumer surplus. Therefore, there is a consumer welfare loss of $\$ 2,700(1,650-$ $4,350)$. The increase in producer surplus is $\$ 1,800(\Delta \pi)$. The net reduction in welfare under price discrimination is $\$ 900(2,700-1,800)$, or an estimate of the increase in the inefficiency of monopoly under price discrimination.

## 5. Can Price Discrimination Lead to Welfare Gains?

Third-degree price discrimination might not necessarily result in a welfare loss when compared with a single-price monopoly. It is possible, however, that price discrimination may lead to a welfare gain over a common monopoly, although it may not yield the socially-optimal amount. Price discrimination in these kinds of cases is desirable because it comes closer to the social optimum than a single-price monopoly does.

To illustrate this idea, let us solve the following optimization problem
optimize

$$
\begin{gather*}
\frac{\left(p_{1}-40\right)\left(260-q_{1}\right)}{2}+\frac{\left(p_{2}-40\right)\left(140-q_{2}\right)}{2}=9000 \\
p_{1}\left(1-\frac{1}{\frac{p_{1}}{q_{1}}}\right)=p_{2}\left(1-\frac{1}{\frac{p_{2}}{q_{2}}}\right)  \tag{18}\\
q_{1}=300-p_{1} \\
q_{2}=180-p_{2} \\
q_{1}, q_{2}, p_{1}, p_{2} \geq 0
\end{gather*}
$$

Compared to Table 3, in (18), we have considered a reduction in the deadweight loss of $\$ 1,000$ (or an increase in the social welfare of $\$ 1,000$ ), establishing a value of 9,000 in the objective function. Using Solver in Excel, we get the following solutions: $q_{1}=140 ; q_{2}=80 ; p_{1}=160 ; p_{2}=$ 100.

These solutions show that a necessary condition for deadweight loss to be reduced is that total output increases. Now, the total output is 220 units. Price discrimination causes the strong market's output reduction to be less than the weak market's output increase ( -20 units vs. +40 units). Accordingly, aggregate output rises by 20 units. Since the reduction in quantity in the strong market is sufficiently small relative to the weak market, welfare rises. The welfare gain is $\$ 1,000$, and the economic profit is still higher $(\$ 21,600)$. Compare the results in Table 3 and expression (9).

From our results, one sees that there is scope for intervention by the regulator, even if s/he has incomplete information. The level of total output can be a useful measure of market performance, and the regulator can use changes in total output to evaluate welfare-maximizing policies (Katz, 1983). Our result is coherent with previous works. Professor Schmalensee (1981) demonstrated that output expansion is only a necessary but not sufficient condition for welfare expansion.

## 6. Case Studies and Theory

### 6.1 First Case

Q. If the monopoly of the previous sections had a total cost function of $\mathrm{TC}=100 Q$, should the regulator (governmental authority) allow or not third-degree price discrimination? Let us use the same inverse demand curves for both groups.
A. The previous analysis assumed that the same sub-markets are served without and with price discrimination. This may not be true. If price discrimination is not allowed, now only the first sub-market would be served with a uniform price of $\$ 200$. That would imply selling 100 units and making a profit of $\$ 10,000$. In this situation, however, it would be desirable to allow price discrimination in order for the firm to reach the second group of consumers as well. Third-degree price discrimination enhances not only the firm's profit but also the total consumer surplus. In fact, the opening of a new sub-market creates a consumer surplus of $\$ 800$ and a profit of $\$ 1,600$ in submarket 2 ; as a result, price discrimination increases welfare by $\$ 2,400$. Welfare in sub-market 1 is unaffected (price and quantity don't change). The total output $Q^{*}$ would increase to 140 units.

### 6.2 Second Case

Q. In a small town, there is only a movie theater whose owner doubts charging a single price per admission or charging different prices to adolescents (group 1) and adults (group 2), with the respective demand functions

$$
\left\{\begin{array}{l}
q_{1}=225-50 p_{1}  \tag{19}\\
q_{2}=105-10 p_{2}
\end{array}\right.
$$

If the total cost function is $\mathrm{TC}=0.5 Q$, what decision should the owner take if his or her goal is profit maximization?
A. If price discrimination is allowed and the firm can prevent the resale of admission tickets between the two groups, the owner should discriminate prices charging teenagers $\$ 2.50$ per person and $\$ 5.50$ per adult. The profit per time period is $\$ 450$, which is greater than $\$ 375$ of profit charging the same price of $\$ 3$ to all customers (both groups). This case shows that a monopolist that faces two different linear demand curves in two sub-markets with the same constant marginal cost in each sub-market sells the same output without and with price discrimination, and price discrimination lowers total welfare. In this case, 150 movie tickets were sold, resulting in a reduction of welfare by $\$ 37.5$.

## 7. Conclusion

Third-degree price discrimination is the practice of charging different prices to two or more different buying groups for the same good. To engage in third-degree price discrimination, the firm first must have market power (the ability to charge prices above marginal cost). Second, the producer must be able to separate customers into distinct markets (groups of consumers with different price elasticities of demand). And, third, the firm must prevent the reselling of the product from customers in one market to customers in another market (arbitrage). In this article, we have seen that third-degree price discrimination enhances the firm's profit. The firm would charge two different prices, high and low, to consumers characterized by a high and low willingness to pay, respectively. However, if the firm is not allowed to discriminate (that is, under a uniform price regime), it would charge a single price to both classes of consumers.

This paper also shows that when an unregulated monopolist attempts to maximize profits, it results in a misallocation of resources. We have used a deadweight loss measure of this inefficiency. The welfare loss from a monopoly arises because of the lower output and higher prices under a monopoly. The monopoly output is lower than the perfectly competitive output. The monopoly price is higher than the perfectly competitive price. In addition, the paper proves that when demand curves are linear and the marginal cost is constant (providing all markets are served), price discrimination has no effect on aggregate output and causes welfare to fall. Robinson (1933) was perhaps one of the first economists to study the effect of discrimination on total output. Since her work, it has been well known that when all sub-markets are served under uniform pricing, price discrimination must decrease social welfare unless aggregate output increases. However, the problem with the output test is that it does not always produce conclusive results (Cowan, 2007).

The implications of price discrimination shown in this paper cannot be generalized. Our discussion and review allow for policy recommendations regarding the treatment of price discrimination. There is no justification for public policies that prohibit price discrimination in general since the welfare effects of price discrimination are ambiguous. Moreover, this paper does not provide the whole story about the effects of third-degree price discrimination. We have
discussed the desirability of price discrimination from the point of view of the efficiency criterion. Nonetheless, price discrimination may be socially justified on the grounds of equity. Under price discrimination, when the price is raised for the "rich" and lowered for the "poor," it has a redistributive outcome; the poor are benefited at the expense of the rich. The essence of the regulator's problem lies in the information structure of the environment.

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